1. (a) $2\sin^2 x - \sqrt{2}\sin x = 0 \implies \sqrt{2}\sin x\left(\sqrt{2}\sin x - 1\right) = 0.$

   Then either $\sin x = 0$ which implies that $\boxed{x = 0, \pi, 2\pi,}$ or $\sin x = \dfrac{1}{\sqrt{2}}$ which implies that $\boxed{x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}.}$

   (b)

$$\begin{aligned} |5x-3| &< 22 \\ -22 < 5x-3 &< 22 \\ -19 < 5x &< 25 \\ -19/5 < x &< 5 \end{aligned}$$

   (c)

$$\begin{aligned} \lim_{x\to 0}\frac{4x}{8+2f(x)} &= -2 \\ \frac{\lim_{x\to0} 4x}{\lim_{x\to0} 8 + \lim_{x\to0} 2f(x)} &= -2 \\ \frac{0}{8+2\lim_{x\to0} f(x)} &= -2 \\ -16 - 4\lim_{x\to0} f(x) &= 0 \\ \lim_{x\to0} f(x) &= \boxed{\text{-4}} \end{aligned}$$

   (d) If x is in the first quadrant, equivalent to $\dfrac{\pi}{6}$, then $\boxed{\tan x = \dfrac{1}{\sqrt{3}},\quad \sec x = \dfrac{2}{\sqrt{3}}.}$

   If x is in the second quadrant, equivalent to $\dfrac{5\pi}{6}$, then $\boxed{\tan x = -\dfrac{1}{\sqrt{3}},\quad \sec x = -\dfrac{2}{\sqrt{3}}.}$

2. (a)

$$\begin{aligned} \lim_{x\to0}\frac{\tan 5x}{x(x-3)} &= \lim_{x\to0}\frac{\sin 5x}{x(x-3)(\cos 5x)} = \lim_{x\to0}\frac{\sin 5x}{5x}\cdot\lim_{x\to0}\frac{5}{(x-3)(\cos 5x)} \\ &= \lim_{x\to0}\frac{5}{(x-3)(\cos 5x)} = \boxed{-\frac{5}{3}.} \end{aligned}$$

   (b) $\displaystyle\lim_{x\to0}\frac{\sec^3 4x}{x^2-4} = \lim_{x\to0}\frac{1}{(\cos^3 4x)(x^2-4)} = \boxed{-\frac{1}{4}.}$

   (c) $\displaystyle\lim_{x\to2}\frac{\frac{1}{x}-\frac{1}{2}}{4-x^2} = \lim_{x\to2}\frac{\frac{2-x}{2x}}{(2+x)(2-x)} = \lim_{x\to2}\frac{2-x}{2x(2+x)(2-x)} = \lim_{x\to2}\frac{1}{2x(2+x)} = \boxed{\frac{1}{16}.}$

3. (a)

$$\begin{aligned} y &= \tan^3(\sin(1-2t)) \\ y' &= 3\tan^2(\sin(1-2t))\sec^2(\sin(1-2t))\cos(1-2t)(-2) \\ &= \boxed{-6\tan^2(\sin(1-2t))\sec^2(\sin(1-2t))\cos(1-2t).} \end{aligned}$$

(b)

$$\begin{aligned} y &= x^2 \cos\left(x^3\right) \\ y' &= x^2\left(-\sin\left(x^3\right)\right)\left(3x^2\right) + \cos\left(x^3\right)(2x) \\ &= \boxed{-3x^4 \sin\left(x^3\right) + 2x\cos\left(x^3\right).} \end{aligned}$$

(c)

$$\begin{aligned} y &= \left(2 + (\cos 5t)^{1/2}\right)^{1/2} \\ y' &= \frac{1}{2}\left(2 + (\cos 5t)^{1/2}\right)^{-1/2}\left(\frac{1}{2}(\cos 5t)^{-1/2}\right)(5) \\ &= \boxed{\frac{5}{4\sqrt{\cos 5t}\sqrt{2+\sqrt{\cos 5t}}}.} \end{aligned}$$

(e) Use implicit differentiation:

$$\begin{aligned} 2x^3y^2 - 3x + y^3 &= -4 \\ 2x^3(2y)\frac{dy}{dx} + y^2\left(6x^2\right) - 3 + 3y^2\frac{dy}{dx} &= 0 \\ 4x^3y\frac{dy}{dx} + 3y^2\frac{dy}{dx} &= 3 - 6x^2y^2 \\ \frac{dy}{dx} &= \boxed{\frac{3-6x^2y^2}{4x^3y+3y^2}.} \end{aligned}$$

4. (a)

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h} \\ &= \lim_{h\to 0}\frac{\sqrt{2x+2h+1}-\sqrt{2x+1}}{h}\cdot\frac{\sqrt{2x+2h+1}+\sqrt{2x+1}}{\sqrt{2x+2h+1}+\sqrt{2x+1}} \\ &= \lim_{h\to 0}\frac{(2x+2h+1)-(2x+1)}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})} \\ &= \lim_{h\to 0}\frac{2h}{h(\sqrt{2x+2h+1}+\sqrt{2x+1})} \\ &= \lim_{h\to 0}\frac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}} \\ &= \frac{2}{\sqrt{2x+1}+\sqrt{2x+1}} \\ &= \boxed{\frac{1}{\sqrt{2x+1}}.} \end{aligned}$$

(b) At $x = 4$, $y = \sqrt{2(4)+1} = 3$.

The slope of the tangent line is $f'(4) = \frac{1}{3}$. The equation of the tangent line is $\boxed{y = 3 + \frac{1}{3}(x-4).}$

The slope of the normal line is -3. The equation of the normal line is $\boxed{y = 3 - 3(x-4).}$

5. The Intermediate Value Theorem for continuous functions states that if a function $f(x)$ is continuous on $[a, b]$, it takes on every value between $f(a)$ and $f(b)$. To show that $y = \sin x + 4x$ has the value 2 for some x, we need to show that y is continuous on some interval $[a, b]$, where 2 lies between $f(a)$ and $f(b)$.

   The function $y = \sin x + 4x$ is continuous because the function $\sin x$ is continuous for all x and the function $4x$ is continuous for all x.

   Let $[a, b]$ be $[0, \pi]$. Then $f(a) = f(0) = 0$ and $f(b) = f(\pi) = 4\pi$. Since $f(a) < 2 < f(b)$, y does equal 2 for some value in $[0, \pi]$.

6. First we must make $f(x)$ continuous at $x = -2$:

   $$\lim_{x \to -2} 2ax - 4 = 3(-2)^2 + b \implies -4a - 4 = 12 + b \implies -4a = 16 + b.$$

   Next we set the left hand derivative as $x \to -2^-$ equal to the right hand derivative as $x \to -2^+$:

   $$2a = 6x \implies 2a = -12 \implies \boxed{a = -6 \text{ and } b = 8.}$$

7. (a) $y' = 12x^2 - 4x^3 = 4x^2(3 - x)$
   $y'' = 24x - 12x^2 = 12x(2 - x)$

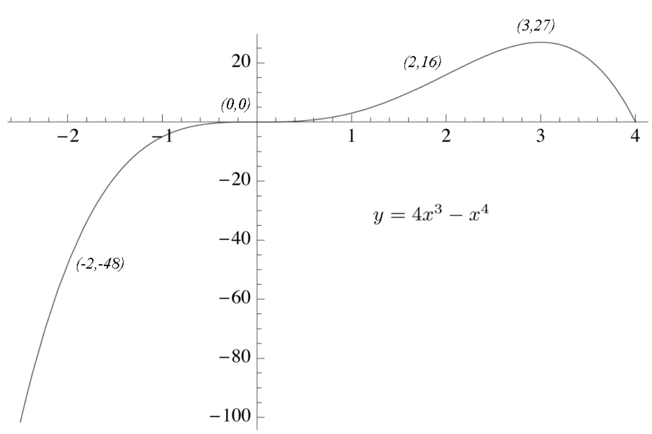
   (b) Critical points for y': $x = 0, 3$. Local maximum at $x = 3$.

y':	+++	|	+++	|	−−−
	incr	0	incr	3	decr

   (c) Critical points for y'': $x = 0, 2$. Inflection points at $x = 0, 2$.

y":	−−−	|	+++	|	−−−
	down	0	up	2	down

   (d)



   (e) Absolute extrema exist because y is continuous on $[-2, 3]$. In addition to the critical points $(0, 0)$, $(2, 16)$, $(3, 27)$, we also check the endpoint $\boxed{(-2, -48),}$ which is the absolute minimum value. The absolute maximum value is $\boxed{(3, 27)\,.}$

7. Let y represent the distance between Car A and the intersection, x represent the distance between the police car and the intersection, and s represent the distance between Car A and the police car. Given $dy/dt = 50$ and $dx/dt = -70$, we wish to find ds/dt when $y = 3$ and $x = 4$. Note that $s = 5$ then.

$$
\begin{aligned}
s^2 &= x^2 + y^2 \\
2s\frac{ds}{dt} &= 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \\
s\frac{ds}{dt} &= x\frac{dx}{dt} + y\frac{dy}{dt} \\
5\frac{ds}{dt} &= 4(-70) + 3(50) \\
\frac{ds}{dt} &= \frac{-280+150}{5} = -\frac{130}{5} = \boxed{\text{-26 mph.}}
\end{aligned}
$$

8. The maximum increase for the function f occurs when $f'(x) = 10$ for all x in $[-1, 7]$. Then $\frac{\Delta f}{\Delta x} = 10 \implies \Delta f = 10(\Delta x) \implies \Delta f = 10(7-(-1)) \implies \Delta f = \boxed{80.}$