APPM 1340 **Final Exam** **December 2007**

On the front of your blue book write: your name, your student ID and a grading table. Please work all of the problems on the exam and box answers if possible. A correct answer with no work will receive little if any credit, while an incorrect answer with some correct explanations and work will receive partial credit. No notes, textbooks or calculators are allowed.

1. (5 points each) Solve the following:

a) $2\sin^2 x - \sqrt{2}\sin x = 0$ on $[0, 2\pi]$

b) $|5x - 3| < 22$

c) Find the $\lim_{x\to 0} f(x)$, if $\lim_{x\to 0} \frac{4x}{8 + 2f(x)} = -2$

d) If I know that $\sin x = \frac{1}{2}$, what are the values of tan x and sec x?

2. (10 points each) Find the following limits:

a) $\lim_{x\to 0} \frac{\tan 5x}{x(x-3)}$

b) $\lim_{x\to 0} \frac{\sec^3 4x}{x^2 - 4}$

c) $\lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{4 - x^2}$

3. (10 points each) Find the following derivatives (use rules and do not simplify):

a) $y = \tan^3[\sin(1 - 2t)]$

b) $y = x^2 \cos(x^3)$

c) $y = \sqrt{2 + \sqrt{\cos 5t}}$

e) $2x^3y^2 - 3x + y^3 = -4$

4. (10 points each)

a) Write the definition of a derivative and then find dy/dx from the definition where $y = \sqrt{2x + 1}$

b) Write the equations of the tangent and normal lines to the curve in 4a when x = 4.

5. (10 points) Does the equation $y = \sin x + 4x$ ever have the value $y = 2$? If it does, state the theorem that helps you determine that, show the conditions are fulfilled and prove that the function does have the value 2 somewhere on its domain.

6. (15 points) In order for the following function to be differentiable at -2, what would the values of a and b have to be?

$$f(x) = \begin{cases} 2ax - 4 & x < -2 \\ 3x^2 + b & x \geq -2 \end{cases}$$

7. (10 points each) Consider the function $y = 4x^3 - x^4$

a) First the first and second derivatives

b) Construct a first derivative chart and find all critical points, label local max and mins

c) Construct a second derivative chart, find all critical points, label any inflection points

d) Draw an accurate graph of the function

e) On the interval [-2,3] find the absolute max and mins of the function if they exist.

7. (15 points) Car A is traveling north at 50 mph away from an intersection. At the same time a police car is traveling west toward the intersection at 70 mph. At the moment when Car A is 3 miles from the intersection and the police car is 4 miles from the intersection, how fast is the distance between the two cars changing?

8. (10 points) If you know that $f'(x) \leq 10$ for all x in the domain of a continuous function, what is the most that the graph of this function could increase on [-1,7]? Explain (you can use a picture to demonstrate what you mean).