APPM 1340 Final Exam Solutions
Fall 2008

1. (a) $\frac{1}{x}$ (b) $|x|$ (c) x^2 (d) x^3 (e) $\sqrt{x}$

2. (a) $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$
   (b) $\cos(\frac{2\pi}{3}) = \frac{-1}{2}$
   (c) $\tan(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}(-2) = -\sqrt{3}$
   (d) $\csc(\frac{2\pi}{3}) = \frac{2}{\sqrt{3}}$
   (e) $\sec(\frac{2\pi}{3}) = -2$
   (f) $\cot(\frac{2\pi}{3}) = \frac{-1}{\sqrt{3}}$

3. (a) $\sqrt{x+1} + 5$
   (b) $\sqrt{x+1} - (x+5)$
   (c) $(x+5)(\sqrt{x+1})$
   (d) $$\begin{aligned} g(f(x)) &= \sqrt{(x+5)+1} = \sqrt{x+6} \\ g(f(10)) &= \sqrt{16} = 4 \end{aligned}$$

4. (a) no.
   (b) yes.
   (c) no. $\lim_{x\to c} f(x) \neq f(c)$
   (d) yes. $\lim_{x\to c} f(x) = f(c)$
   (e) no. $\lim_{x\to c^-} f'(x) \neq \lim_{x\to c^+} f'(x)$

5. $\lim_{x\to x_0} f(x) = L$
   For every $\epsilon > 0$ there is a $\delta > 0$ such that $|x - x_0| < \delta$ guarantees $|f(x) - L| < \epsilon$.

6. (a) $4 + 14 + 1 = \boxed{19}$

(b)

$$
\begin{aligned}
& \lim_{x\to 1} \frac{x-1}{(\sqrt{x+3}-2)} \cdot \frac{(\sqrt{x+3}+2)}{(\sqrt{x+3}+2)} \\
= & \lim_{x\to 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x+3)-4} \\
= & \lim_{x\to 1} \sqrt{x+3}+2 \\
= & \boxed{4}
\end{aligned}
$$

(c)

$$
\begin{aligned}
& \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} \\
= & \lim_{h\to 0} \frac{x^2+2hx+h^2-x^2}{h} \\
= & \lim_{h\to 0} \frac{2hx+h^2}{h} \\
= & \lim_{h\to 0} 2x+h \\
= & \boxed{\text{2x}}
\end{aligned}
$$

(d)

$$
\begin{aligned}
& \lim_{x\to 0} \frac{x+x\cos(x)}{\sin(x)\cos(x)} \\
= & \lim_{x\to 0} \frac{x}{\sin(x)\cos(x)} + \frac{x\cos(x)}{\sin(x)\cos(x)} \\
= & \lim_{x\to 0} \frac{1}{\cos(x)} \cdot \frac{x}{\sin(x)} + \frac{x}{\sin(x)} \\
= & 1+1 \\
= & \boxed{2}
\end{aligned}
$$

(e)

$$\begin{aligned}
&\lim_{x\to 0}\frac{x\csc(2x)}{\cos(5x)}\\
=\;&\lim_{x\to 0}\frac{1}{\cos(5x)}\cdot\frac{x}{\sin(2x)}\\
=\;&\lim_{x\to 0}\frac{1}{\cos(5x)}\cdot\frac{2x}{\sin(2x)}\cdot\frac{1}{2}\\
=\;&1\cdot 1\cdot \tfrac{1}{2}\\
=\;&\boxed{\tfrac{1}{2}}
\end{aligned}$$

7. (a) $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

(b) i.

$$\begin{aligned}
&\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}\\
=\;&\lim_{h\to 0}\frac{3(x^2+2hx+h^2-x^2)}{h}\\
=\;&\lim_{h\to 0}3(2x+h)\\
=\;&3\cdot 2x\\
=\;&\boxed{6\text{x}}
\end{aligned}$$

ii.

$$\begin{aligned}
&\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sqrt{x+h+1}}-\frac{1}{\sqrt{x+1}}\right)\\
=\;&\lim_{h\to 0}\frac{1}{h}\left(\frac{\sqrt{x+1}-\sqrt{x+h+1}}{\sqrt{x+h+1}\sqrt{x+1}}\cdot\frac{(\sqrt{x+1}+\sqrt{x+h+1})}{(\sqrt{x+1}+\sqrt{x+h+1})}\right)\\
=\;&\lim_{h\to 0}\frac{1}{h}\left(\frac{(x+1)-(x+h+1)}{\sqrt{x+h+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+h+1})}\right)\\
=\;&-\frac{1}{(x+1)\cdot 2\sqrt{x+1}}\\
=\;&\boxed{-\tfrac{1}{2}(x+1)^{-\frac{3}{2}}}
\end{aligned}$$

8. (a) $y' = 2x+1$

(b) $w' = -6z^{-3}+z^{-2}$

(c) $u = \frac{5}{2}x^{1/2}+\frac{1}{2}x^{-1/2} \quad\Longrightarrow\quad u' = \frac{5}{4}x^{-1/2}-\frac{1}{4}x^{-3/2}$

(d) $y' = -4x^{-5}\sin(x)\tan(x)+x^{-4}\cos(x)\tan(x)+x^{-4}\sin(x)\sec^2(x)$

(e) $g'(\theta) = \frac{1}{3}(1+\cot(2\theta))^{-2/3}\left(-\csc^2(2\theta)\right)(2) = -\frac{2}{3}\csc^2(2\theta)(1+\cot(2\theta))^{-2/3}$

9. (a) $\sin(2y) + x\cos(2y)(2)\dfrac{dy}{dx} = \dfrac{dy}{dx}\cos(2x) + y(-\sin(2x))(2)$

$\dfrac{dy}{dx}(\cos(2x) - 2x\cos(2y)) = \sin(2y) + 2y\sin(2x)$

$\dfrac{dy}{dx} = \boxed{\dfrac{\sin(2y) + 2y\sin(2x)}{\cos(2x) - 2x\cos(2y)}}$

(b) At point $(\pi/4, \pi/2)$,

$\dfrac{dy}{dx} = \dfrac{\sin(\pi) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)}{\cos\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right)\cos(\pi)} = \dfrac{0+\pi}{0+\frac{\pi}{2}} = \boxed{2}.$

The equation of the tangent line is

$y = m(x - x_1) + y_1$

$\boxed{y = 2\left(x - \dfrac{\pi}{4}\right) + \dfrac{\pi}{2}}$

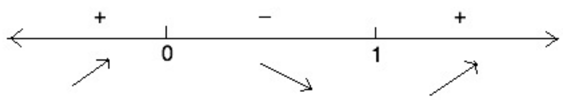
10.

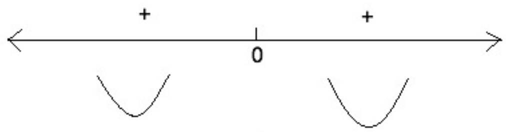
$$y = 2x - 3x^{2/3}$$
$$y' = 2 - 3(\frac{2}{3})x^{-1/3} = 2 - 2x^{-1/3}$$
$$y'' = 2(\frac{1}{3})x^{-4/3} = \frac{2}{3}x^{-4/3}$$

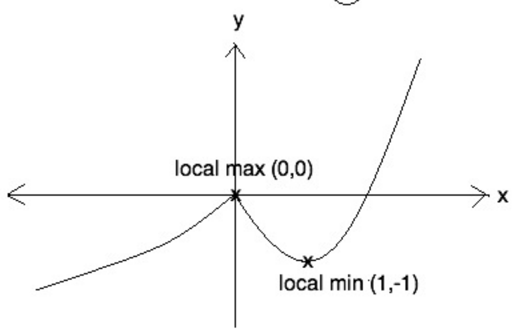
C.P.s: y' DNE at $x = 0$.
$y' = 0 \Longrightarrow 2 - 2x^{-1/3} = 0 \Longrightarrow x = 1$,
so $x = 0, 1$ are C.P.

P.I.P.s: y'' DNE at $x = 0$.
$y'' = 0$ for no values of x.

$$y'(-1) = 2 - 2(-1) = 2 + 2 > 0$$
$$y'(1/2) = 2 - \frac{2}{\sqrt[3]{\frac{1}{2}}} = 2 - 2\cdot\sqrt[3]{2} < 0$$
$$y'(2) = 2 - \frac{2}{\sqrt[3]{2}} > 0$$
$$y''(-1) = \frac{2}{3}(-1)^{4/3} > 0$$
$$y''(1) = \frac{2}{3}(1) > 0$$
$$y(0) = 0$$
$$y(1) = 2 - 3 = -1$$







11. (a) The ratio of the radius to the height is

$$\frac{r}{h} = \frac{4}{12} = \frac{1}{3}.$$

Therefore $r = h/3$. When the height is 9 inches, the radius is 3 inches.

$$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\frac{1}{9}h^3 = \frac{\pi}{27}h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{9}\cdot 9^2\left(-\frac{1}{6}\right)$$

$$\frac{dV}{dt} = -\frac{9}{6}\pi = \boxed{-\frac{3}{2}\pi \text{ m}^3/\text{min}}$$

(b) $V = \frac{\pi}{27}h^3 = \frac{\pi}{27}9^3 = 27\pi$

$$t = \frac{27\pi}{\frac{3}{2}\pi} = \frac{27 * 2}{3} = \boxed{18 \text{ min}}$$

12. (a)
$$\begin{aligned} s(t) &= 179 - 16t^2 \\ v(t) &= -32t \\ a(t) &= -32 \end{aligned}$$

Speed is $32t$.

(b)
$$\begin{aligned} s(t) &= 0 \\ 16t^2 &= 179 \\ t &= \tfrac{\sqrt{179}}{4}\text{sec} \end{aligned}$$

(c) $v(\frac{\sqrt{179}}{4}) = -32 \cdot \frac{\sqrt{179}}{4} = \boxed{-8\sqrt{179}\text{ m/sec}}$

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