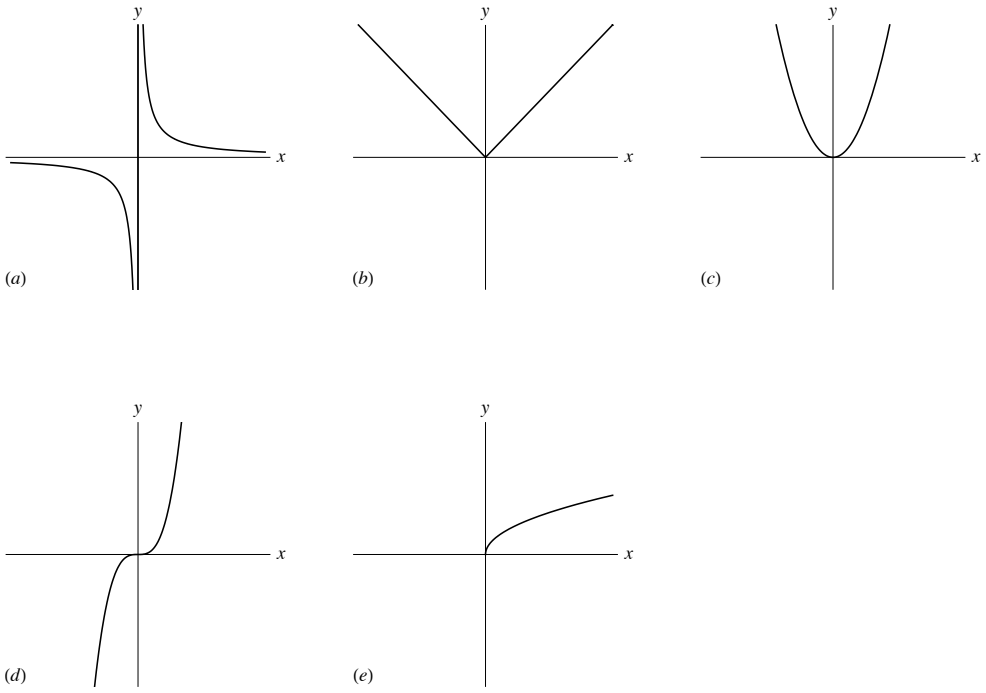
APPM 1340 — Final — December 15, 2008

On the front of your bluebook print (1) your name, (2) your student ID number, and (3) a grading table. **Explain all of your answers.** A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. The exam is out of 200 points but there are 240 available points so it is possible to earn up to 40 extra credit points. No electronic devices of any kind (e.g. cell phones, calculators, etc.) are permitted.

1. (15 points) Match the functions $y = x^2$, $y = \sqrt{x}$, $y = |x|$, $y = 1/x$ and $y = x^3$ with their graphs pictured below.



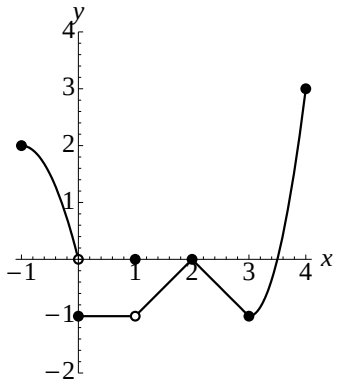
2. (20 points) Find the value of all six trig functions for the angle $\theta = \frac{2\pi}{3}$.

(a) $\sin \frac{2\pi}{3}$ (b) $\cos \frac{2\pi}{3}$ (c) $\tan \frac{2\pi}{3}$ (d) $\csc \frac{2\pi}{3}$ (e) $\sec \frac{2\pi}{3}$ (f) $\cot \frac{2\pi}{3}$

3. (12 points) Let $f(x) = x + 5$ and $g(x) = \sqrt{x+1}$. Find each of the following:

   (a) $(f \circ g)(x)$
   (b) $(g - f)(x)$
   (c) $(f \cdot g)(x)$
   (d) $g(f(10))$

4. (20 points) Answer the following questions about the function $f(x)$ whose graph is pictured below.



   (a) Does $\lim_{x\to 0} f(x)$ exist?

   (b) Does $\lim_{x\to 1} f(x)$ exist?

   (c) Is $f(x)$ continuous at $x = 1$? Explain why or why not.

   (d) Is $f(x)$ continuous at $x = 2$? Explain why or why not.

   (e) Is $f(x)$ differentiable at $x = 2$? Explain why or why not.

5. (10 points) Using ϵ and δ state the precise definition of $\lim_{x\to x_0} f(x) = L$.

6. (35 points) Calculate each of the following limits or explain why it does not exist.

   (a) $\lim_{x\to 2} x^2 + 7x + 1$

   (b) $\lim_{x\to 1} \dfrac{x-1}{\sqrt{x+3}-2}$

   (c) $\lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$

   (d) $\lim_{x\to 0} \dfrac{x + x\cos x}{\sin x \cos x}$

   (e) $\lim_{x\to 0} \dfrac{x \csc 2x}{\cos 5x}$

7. (a) (6 points) State precise definition of the derivative of the function $f(x)$ with respect to the variable x

   (b) Use the definition of the derivative to find $f'(x)$ for the following functions. Be sure to show all your work.

      i. (7 points) $f(x) = 3x^2$

      ii. (10 points) $f(x) = \frac{1}{\sqrt{x+1}}$

8. (20 points) Find the derivative of the following functions using any method we have learned.

   (a) $y = x^2 + x + 8$

   (b) $w = 3z^{-2} - \frac{1}{z}$

   (c) $u = \frac{5x+1}{2\sqrt{x}}$

   (d) $y = x^{-4} \sin x \tan x$

   (e) $g(\theta) = \sqrt[3]{1 + \cot(2\theta)}$

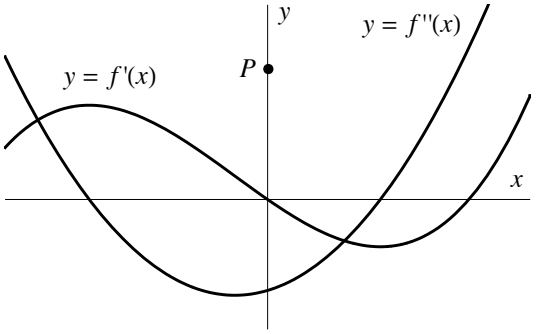
9. (15 points) Implicit Differentiation

   (a) For the equation $x \sin(2y) = y \cos(2x)$, use implicit differentiation to find $\frac{dy}{dx}$.

   (b) The point $(\pi/4, \pi/2)$ lies on the curve of the above equation. Find the equation of the line tangent to the curve at the given point.

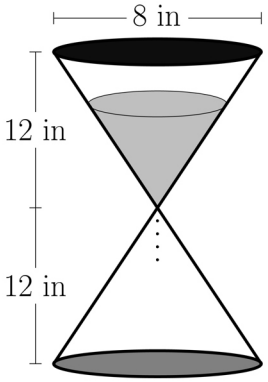
10. (20 points) Let $y = 2x - 3x^{2/3}$. Use both y' and y'' to draw a graph of the function y. Your final graph should include the coordinate axes. Label all local extrema and inflection points.

11. (20 points) Pictured below are the first and second derivative graphs of a function $y = f(x)$.



   (a) Using these graphs as a guide, sketch the general shape of the original function $y = f(x)$.

   (b) Now position your sketch on the axes containing f' and f'' so that it passes through the point P. (Please trace the graph containing f' and f'' into your bluebook.)



12. (15 points) The Wicked Witch of the West has imprisoned Dorothy in her castle, hoping to steal Dorothy's ruby red slippers. The witch turns over her sand-filled hourglass and says, "That's how much longer you've got to be alive. And it isn't long, my pretty. It isn't long!"

    (a) Each half of the hourglass is shaped like a cone, 12 in. high and 8 in. across at the base. If the sand level in the upper half is dropping at a rate of $\frac{1}{6}$ in/min when the sand is 9 in deep, how fast is the sand draining? (When calculating you may assume the two cones meet at a point.)

    (b) If the sand drains at a constant rate, how many minutes does Dorothy have to live?

13. (15 points) Had Galileo dropped a cannonball from the tower of Pisa, 179 ft above the ground, the ball's height above ground t seconds into the fall would have been $s = 179 - 16t^2$.

    (a) What would have been the ball's velocity, speed and acceleration at time t?

    (b) About how long would it have taken the ball to hit the ground?

    (c) What would have been the ball's velocity at the moment of impact?