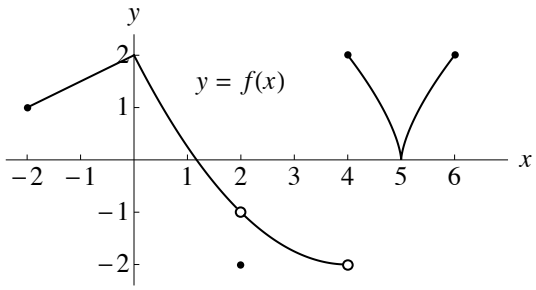
APPM 1340 **Final Exam** **Fall 2009**

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) section number, and (3) a grading table on the front of your bluebook. **Start each problem on a new page. Simplify your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Unless otherwise indicated, **show all work**. There are **20 extra credit points** in this exam.



1. (15 points) The function $f(x)$ is defined on $[-2, 6]$. Refer to the graph of $f(x)$ above to answer the following questions. No explanations are necessary.

   (a) $\lim_{x\to 2} f(x) =$

   (b) $\lim_{x\to 4} f(x) =$

   (c) $\lim_{x\to 5} f(x) =$

   (d) $\lim_{x\to 6^-} f(x) =$

   (e) $\lim_{x\to 6^+} f(x) =$

   (f) For what values of x is f not continuous?

   (g) For what values of x is f not differentiable?

2. (20 points) Let $f(x) = 1 + \sqrt{3x - 6}$ and $g(x) = x + 2$.

   (a) Find the composite function $(f \circ g)(x)$.

   (b) Use the definition of derivative to find $(f \circ g)'$.

3. (15 points) Match the following functions to the graphs in Figure 1. No explanation is necessary.

(a) $y = \left|x^2 - 1\right|$ (c) $y = \sqrt{1-x}$ (e) $y = -\sin x$

(b) $y = \left|-x^2 - 1\right|$ (d) $y = \sqrt[3]{x}$

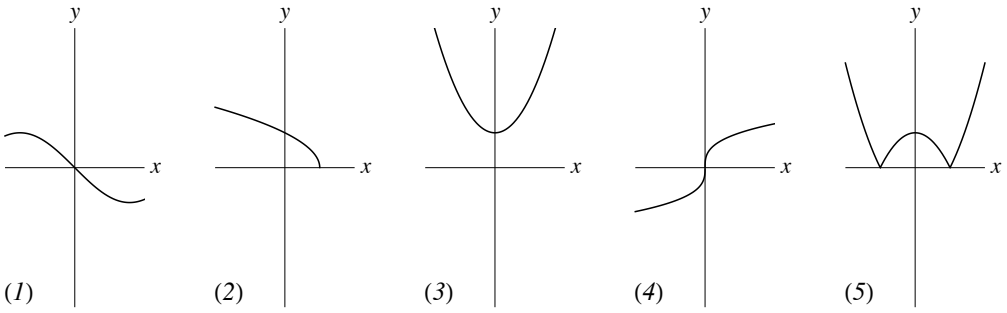


Figure 1: Functions

4. (15 points) Match the graphs of the functions in Figure 1 to their derivatives in Figure 2. No explanation is necessary.

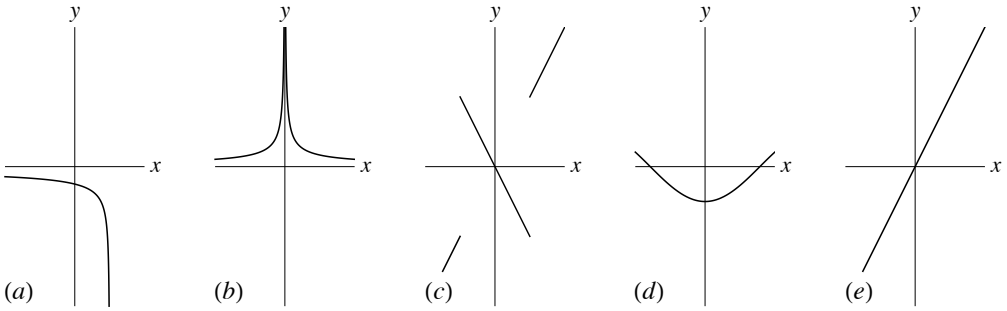


Figure 2: Derivatives

5. (15 points) For the following statements, answer TRUE if the statement is always true. Otherwise sketch a counterexample. No explanation is necessary.

   (a) If $f(x)$ is continuous on $[-3, -1]$ and has a local maximum value at $x = -2$, then $f'(-2) = 0$.

   (b) If an even function $f(x)$ has a local maximum value at $x = a$, then f has a local maximum at $x = -a$.

   (c) If an odd function $f(x)$ is decreasing on $[-a, 0]$, then f is increasing on $[0, a]$.

6. (25 points) Evaluate the following limits.

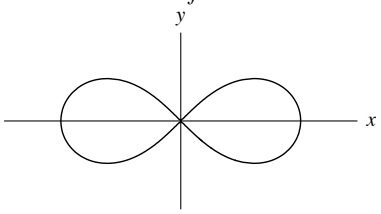
   (a) $\lim_{x \to 7^+} \dfrac{7-x}{x^2-49} =$

   (b) $\lim_{t \to 5^-} \dfrac{t^2-3t-4}{t-5} =$

   (c) $\lim_{\theta \to 0} 2\theta \sec 2\theta \tan 3\theta =$

7. (10 points)

Lemniscate of Bernoulli



$$(x^2+y^2)^2 = 2(x^2-y^2)$$

Find dy/dx. You need not simplify.

8. (25 points) Let

$$y = \frac{x+x^2}{1-4x^2}.$$

   (a) Find the horizontal and vertical asymptotes of the graph of y.

   (b) Find an equation for the line tangent to the curve at $x = -1$.

9. (30 points) Let

$$y = \tan x - x, \quad -\tfrac{\pi}{2} < x < \tfrac{\pi}{2}.$$

Use y' and y'' to sketch the general shape of the graph of y. Label all local extrema and inflection points.

10. (10 points) Let

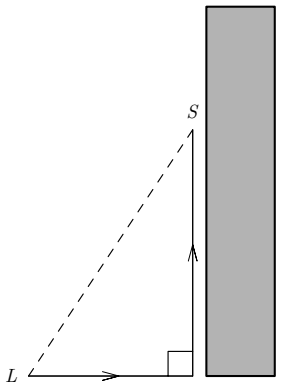
$$g(x) = \begin{cases} x^2 \cos\left(\frac{1}{x^3}\right), & x \neq 0 \\ 1, & x = 0. \end{cases}$$

Is g continuous at $x = 0$? Justify your answer.

11. (40 points) Superman has arranged to meet Lois Lane on the rooftop of the Daily Planet building. A few minutes before midnight he takes off vertically from the front entrance of the 900-foot-high building. Let

$$s(t) = 240t - 16t^2$$

represent Superman's height in feet above the ground after t seconds. (Assume gravity is the only force acting on him after takeoff.)



(a) What is Superman's initial velocity?

(b) How high above the ground is Superman after 5 seconds?

(c) How fast is he flying then?

(d) At this moment Lois is down the street, 600 feet from the Daily Planet building, driving at a constant speed of 75 feet/sec toward the front entrance. Is the distance between Lois and Superman increasing or decreasing then? How fast is it changing?

(e) What is the maximum height Superman can reach given his initial velocity?

(f) Will Lois arrive at the front entrance before Superman reaches the rooftop?