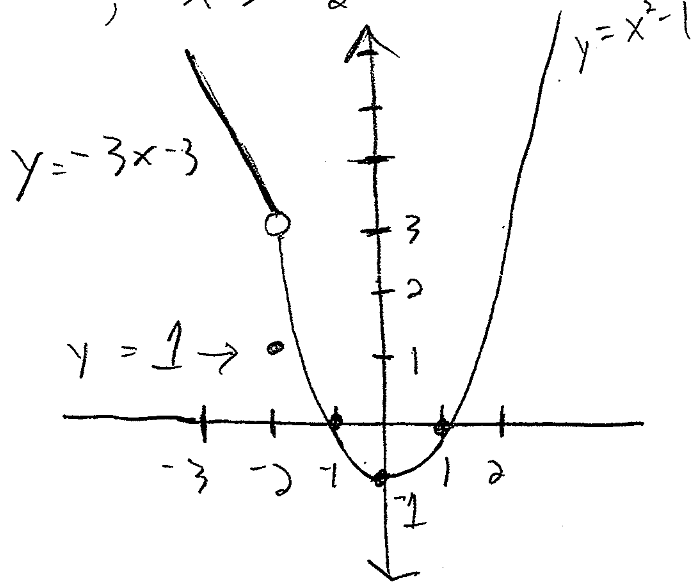
1) $f(x) = \begin{cases} -3x-3, & x<-2 \\ 1, & x=-2 \\ x^2-1, & x>-2 \end{cases}$

a)

x-intercepts $(-1,0), (1,0)$

y-intercepts $(0,-1)$



b) $\lim_{x \to -2} f(x) = 3$

As we approach $x=-2$ from both the left and right hand sides the function value gets arbitrarily close to 3. (Although $f(-2) = 1$)

c) Average rate of change of $f(x)$ over $(-2, 1)$

$\frac{\Delta y}{\Delta x} = \frac{3-0}{-2-1} = \frac{3}{-3} = \boxed{-1}$

2. The formal definition of a limit states that

$\lim_{x \to x_0} f(x) = L$ exists and equals L if

$\forall \varepsilon > 0$, $\exists$ corresponding $\delta > 0$ such that

$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon.$

3) Given: $f(x) = \sqrt{x+6}$ $\qquad x_0 = 10$

$L = 4 \qquad \epsilon = \frac{1}{2}$

We begin with $|f(x) - L| < \epsilon$.

$$|\sqrt{x+6} - 4| < \tfrac{1}{2}$$

$$-\tfrac{1}{2} < \sqrt{x+6} - 4 < \tfrac{1}{2}$$

$$3\tfrac{1}{2} < \sqrt{x+6} < 4\tfrac{1}{2} \qquad \text{(add 4)}$$

$$\tfrac{7}{2} < \sqrt{x+6} < \tfrac{9}{2}$$

$$\tfrac{49}{4} < x+6 < \tfrac{81}{4} \qquad \text{(square)}$$

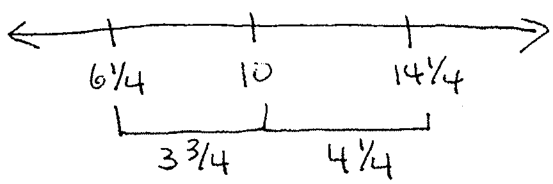
$$\tfrac{25}{4} < x < \tfrac{57}{4} \qquad \text{(subtract 6)}$$

$$6\tfrac{1}{4} < x < 14\tfrac{1}{4}$$

Now we center the interval about $x_0 = 10$.

$$-3\tfrac{3}{4} < x - 10 < 4\tfrac{1}{4} \qquad \text{(subtract 10)}$$

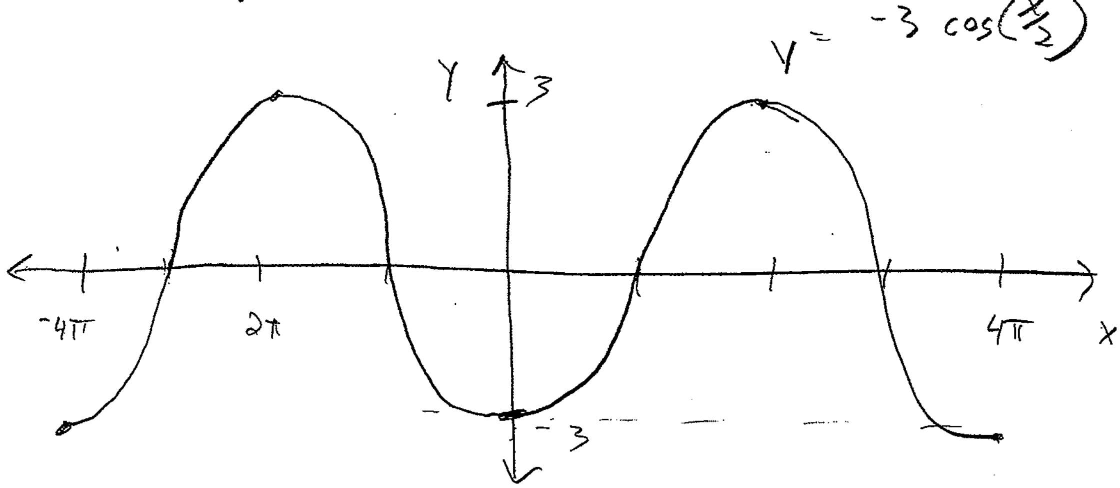
We choose $\boxed{\delta = 3\tfrac{3}{4} = \tfrac{15}{4}}$.



δ is the smaller distance.

4) Sketch $y = -3\cos\left(\frac{x}{2}\right)$

Period 4π



5) Is $y = -3\cos\left(\frac{x}{2}\right)$ even or odd or neither?

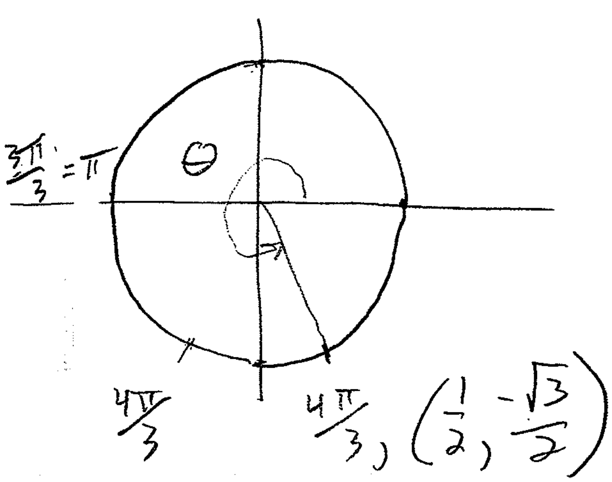
$f(-x) = -3\cos\left(\frac{-x}{2}\right) = -3\cos\left(-\frac{x}{2}\right)$

$\cos\theta$ is even: $\cos(-\theta) = \cos(\theta)$ so

$f(-x) = -3\cos\left(\frac{-x}{2}\right) = -3\cos\left(\frac{x}{2}\right)$

so $f(-x) = f(x)$ so $-3\cos\left(\frac{x}{2}\right)$ is <u>even</u>!

6) $\theta = \frac{5\pi}{3}$



$\sin\theta = y$

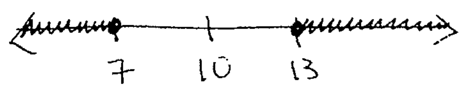
$\cos\theta = x$

$\sin\frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$ $\qquad \csc\frac{5\pi}{3} = \frac{-2}{\sqrt{3}}$

$\cos\frac{5\pi}{3} = \frac{1}{2}$ $\qquad \sec\frac{5\pi}{3} = 2$

$\tan\frac{5\pi}{3} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$ $\qquad \cot\frac{5\pi}{3} = \frac{-1}{\sqrt{3}}$

7) $|x-10| \geq 3$ means the distance from x to 10 on the real number line is greater than or equal to 3.
Therefore $x \geq 13$ or $x \leq 7$.



Case 1 ($x-10$ is pos):

$$x-10 \geq 3$$

$$\boxed{x \geq 13}$$

Case 2 ($x-10$ is neg):

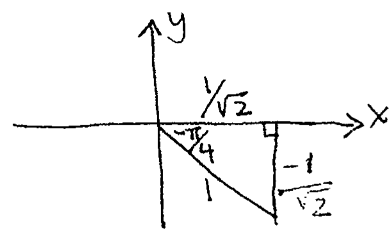
$$x-10 \leq -3$$

$$\boxed{x \leq 7}$$

8) $$\cos^2 \theta = \frac{1+\cos 2\theta}{2}$$

$$\cos^2\left(-\frac{\pi}{8}\right) = \frac{1+\cos 2\left(-\frac{\pi}{8}\right)}{2}$$

$$= \frac{1+\cos\left(-\frac{\pi}{4}\right)}{2}$$



Since $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$,

$$\cos^2\left(-\frac{\pi}{8}\right) = \frac{1+\frac{\sqrt{2}}{2}}{2}$$

$$= \boxed{\frac{2+\sqrt{2}}{4}}$$

9. $P(2,-1)$ $\quad$ $Q(-3,5)$

Line L through P and Q

Slope: $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-1)}{-3-2} = \dfrac{6}{-5} = \boxed{-\dfrac{6}{5}}$

Point-Slope form: $\boxed{y-(-1) = \frac{-6}{5}(x-2)}$ using P

OR $\boxed{y-5 = \frac{-6}{5}(x+3)}$ using Q

OR slope-intercept form: $\boxed{y = \frac{-6}{5}x + \frac{7}{5}}$

Line M $\perp$ to L, through $Q(-3,5)$

Slope: $m_2 = \dfrac{-1}{m_1} = \boxed{\dfrac{5}{6}}$

Point-Slope form: $\boxed{y-5 = \frac{5}{6}(x-3)}$

OR slope-intercept form: $\boxed{y = \frac{5}{6}x - \frac{15}{2}}$

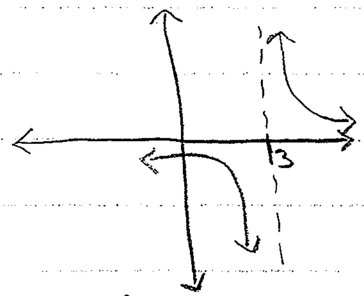
10. a) $\lim_{x \to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2} = \frac{0}{0}$ indeterminate form

so try factoring:

$= \lim_{x \to 0} \frac{\cancel{x^2}(5x+8)}{\cancel{x^2}(3x^2-16)} = \frac{8}{-16} = \boxed{\frac{-1}{2}}$

b) $\lim_{x \to 3^+} \frac{1}{x-3} = \frac{1}{0}$

so is + or - ∞...



Graph looks like ↗

is $\frac{1}{x}$ shifted to the right 3.

So $\lim_{x \to 3^+} \frac{1}{x-3} = \boxed{+\infty}$

c) $\lim_{x \to -5} x^2 - 10x + 1 = (-5)^2 - 10(-5) + 1$

$= 25 + 50 + 1$

$= \boxed{76}$

11. $x^2 + y^2 - 3y + 8x = 0$ complete the square

$x^2 + 8x \quad + y^2 - 3y \quad = 0$

4   -1.5

16   2.25

$x^2 + 8x + 16 + y^2 - 3y + 2.25 = 16 + 2.25$

$(x+4)^2 + (y-1.5)^2 = 18.25$

radius: $\sqrt{18.25}$

center: $(-4, 1.5)$