APPM 1340 **Exam 1 Solutions** **Fall 2009**

1. (a) Graph 4. Left 2, up 2.
   (b) Graph 1. Right 2, down 2.
   (c) Graph 2. Down 2, reflect negative up.
   (d) Graph 3. Reflect down, down 2, reflect graph up.

2.

$$\begin{aligned} \left|6+\frac{4}{x}\right| &< 2 \\ -2 < 6+\frac{4}{x} &< 2 \\ -8 < \frac{4}{x} &< -4 \\ -\frac{1}{8} > \frac{x}{4} &> -\frac{1}{4} \\ -\frac{1}{4} < \frac{x}{4} &< -\frac{1}{8} \\ -1 < x &< -\frac{1}{2} \end{aligned}$$

The solution set is $\boxed{(-1,-1/2)}$. (Note that the $<$ signs flip when we take the reciprocal.)

3. We rewrite the given equation in slope-intercept form.

$$\begin{aligned} 3x-7y &= 1 \\ -7y &= -3x+1 \\ y &= \frac{3}{7}x-\frac{1}{7} \end{aligned}$$

The slope of the given line is $3/7$. The slope of the perpendicular line l is the negative reciprocal, or $-7/3$. An equation for line l using the point-slope formula is

$$\begin{aligned} y &= y_1+m(x-x_1) \\ y &= -3-\frac{7}{3}(x-2) \\ y &= -3-\frac{7}{3}x+\frac{14}{3} \\ y &= -\frac{7}{3}x+\frac{5}{3} \end{aligned}$$

Point (a,b) lies on line l so $\boxed{b=-\frac{7}{3}a+\frac{5}{3}}$.

4. (a) $\lim_{x\to 0} f(x) =$ undefined
   (b) $\lim_{x\to .8} f(x) = -1$
   (c) $\lim_{x\to 1} f(x) =$ undefined
   (d) $\lim_{x\to 2} f(x) = 0$
   (e) $\lim_{x\to 3} f(x) = -1$

5. (a) We substitute the value $x=2$.

$$\lim_{x\to 2}\frac{x^3-4x-1}{x-3} = \frac{2^3-4(2)-1}{2-3} = \frac{8-8-1}{2-3} = \frac{-1}{-1} = \boxed{1}$$

   (b) We factor the denominator and multiply by the conjugate of the numerator. Note that $(1-r)/(r-1)=-1$.

$$\begin{aligned} \lim_{r\to 1}\frac{2-\sqrt{r+3}}{r^2+4r-5}\cdot\frac{2+\sqrt{r+3}}{2+\sqrt{r+3}} &= \lim_{r\to 1}\frac{4-(r+3)}{(r-1)(r+5)(2+\sqrt{r+3})} \\ &= \lim_{r\to 1}\frac{1-r}{(r-1)(r+5)(2+\sqrt{r+3})} \\ &= \lim_{r\to 1}\frac{-1}{(r+5)(2+\sqrt{r+3})} \\ &= \frac{-1}{(6)(2+\sqrt{4})} = \frac{-1}{(6)(2+2)} \\ &= \boxed{-\frac{1}{24}} \end{aligned}$$

(c) We use the Sandwich Theorem.

Since

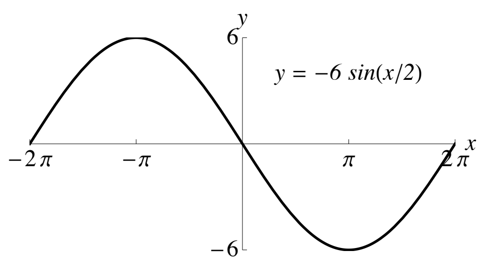
$$\lim_{x\to -1} \sqrt{3-6x-3x^2} = \sqrt{3+6-3} = \sqrt{6}$$

and

$$\lim_{x\to -1} \sqrt{5-2x-x^2} = \sqrt{5+2-1} = \sqrt{6},$$

then $\lim_{x\to -1} g(x) = \boxed{\sqrt{6}}$.

6.



7.

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\sin^2\left(-\frac{\pi}{12}\right) = \frac{1-\cos\left(2\left(-\frac{\pi}{12}\right)\right)}{2}$$

$$= \frac{1-\cos\left(-\frac{\pi}{6}\right)}{2}$$

The angle $-\pi/6$ lies in the fourth quadrant where cosine is positive. Then

$$\cos(-\pi/6) = \cos(\pi/6) = \sqrt{3}/2.$$

Substituting into the equation above, we get

$$\sin^2\left(-\frac{\pi}{12}\right) = \frac{1-\cos\left(-\frac{\pi}{6}\right)}{2}$$

$$= \frac{1-\frac{\sqrt{3}}{2}}{2}$$

$$= \boxed{\frac{2-\sqrt{3}}{4}}.$$

We can find the value of $\cos^2\left(-\frac{\pi}{12}\right)$ using the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$.

$$\cos^2\theta = 1 - \sin^2\theta$$

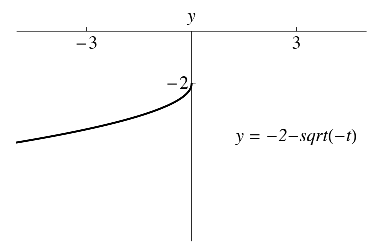
$$\cos^2\left(-\frac{\pi}{12}\right) = 1 - \sin^2\left(-\frac{\pi}{12}\right)$$

$$= 1 - \left(\frac{2-\sqrt{3}}{4}\right)$$

$$= \frac{4-(2-\sqrt{3})}{4}$$

$$= \boxed{\frac{2+\sqrt{3}}{4}}.$$

8.



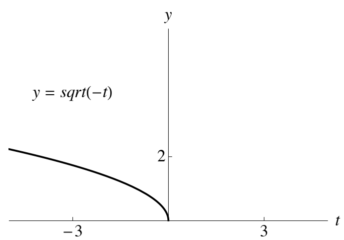
(a) If you graph the function f using translations, you can see that the domain is $(-\infty, 0]$ and the range is $(-\infty, -2]$.

To determine the domain algebraically, we note that the expression under the square root sign must be greater than or equal to 0. If $-t \geq 0$, then $t \leq 0$. The function f is increasing so we can examine the endpoints of the domain to find the range. As t approaches $-\infty$, $f(t)$ also will approach $-\infty$. When $t = 0$, $f(t) = -2$. Therefore the range is $f(t) \leq -2$.

(b)

$$\begin{aligned} g(f(t)) &= g(-2 - \sqrt{-t}) \\ &= -(-2 - \sqrt{-t} + 2) \\ &= -(-\sqrt{-t}) \\ &= \boxed{\sqrt{-t}}. \end{aligned}$$

(c)



A function $g \circ f$ is even if $g(f(-t)) = g(f(t))$, and it is odd if $g(f(-t)) = -g(f(t))$. The function $(g \circ f)(t) = \sqrt{-t}$ is $\boxed{\text{neither even nor odd}}$ because $g(f(-t)) = \sqrt{t}$ does not equal $g(f(t)) = \sqrt{-t}$, and also does not equal $-g(f(t)) = -\sqrt{-t}$. In fact $g(f(-t)) = \sqrt{t}$ is undefined for the domain of f except at $t = 0$.

If you graph the function $g(f(t)) = \sqrt{-t}$, you can see that it is not symmetric about the y-axis and not symmetric about the origin.

Another way to show that $g \circ f$ is neither even nor odd is to observe that the function includes the point $(-1, 1)$ but does not include the reflection of the point across the y-axis $(1, 1)$, and does not include the reflection of the point across the origin $(1, -1)$.

9. If the stock price decreases by $\$13$ per month for the first three months, it decreases a total of $\$39$ for the three months, from a price of $\$129$ to $\$90$. It then increases by $\$90$ over the next 9 months to reach $\$180$, which equals an average rate of change of $\$10$ per month.

More formally, the average rate of change of a function $f(t)$ is

$$\frac{\Delta f}{\Delta t} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}.$$

Let f represent the stock price function and t represent the elapsed time in months. Then $f(3)$ represents the price after 3 months. The average rate of change for the first three months is

$$\begin{aligned} \frac{\Delta f}{\Delta t} = \frac{f(3) - f(0)}{3 - 0} &= -13 \\ \frac{f(3) - 129}{3} &= -13 \\ f(3) - 129 &= -39 \\ f(3) &= 90. \end{aligned}$$

The price after three months is $\$90$. Now we calculate the average rate of change for the remaining 9 months.

$$\begin{aligned} \frac{\Delta f}{\Delta t} &= \frac{f(12) - f(3)}{12 - 3} \\ &= \frac{180 - 90}{9} = \frac{90}{9} \\ &= 10. \end{aligned}$$

The average rate of change for the remaining 9 months is $\boxed{\$10 \text{ per month}}$.

Extra Credit

a.

$$\begin{aligned} \sin(A - B) &= \sin A \cos B - \cos A \sin B \\ \sin(-x) &= \sin(0 - x) \\ &= \sin 0 \cos x - \cos 0 \sin x \\ &= (0)(\cos x) - (1)(\sin x) \\ &= 0 - \sin x \\ &= -\sin x. \end{aligned}$$

b.

$$\begin{aligned}\cos(A-B) &= \cos A\cos B + \sin A\sin B \\ \cos(-x) &= \cos(0-x) \\ &= \cos 0\cos x + \sin 0\sin x \\ &= (1)(\cos x) + (0)(\sin x) \\ &= \cos x + 0 \\ &= \cos x.\end{aligned}$$