APPM 1340 Midterm 2 Solutions Fall 2008

1a) $0 < t < 3$ moving forward when $v > 0$

b) $3 < t < 8$ moving backward when $v < 0$

c) $0 < t < 2,\ 3 < t < 5$ speeding up when $|v|$ increasing

d) $0 < t < 2,\ 7 < t < 8$ acceleration positive when slope > 0

e) $5 \le t \le 7$ greatest speed when $|v|$ is maximized

2a) $f(x)$ is continuous at a point c in its domain when:

1. $f(c)$ exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

2b) The derivative of the function $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ provided the limit exists.

3a) $g(t) = \sqrt{2t-1}$

$$g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2(t+h)-1} - \sqrt{2t-1}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2t+2h-1} - \sqrt{2t-1}}{h} \cdot \frac{\sqrt{2t+2h-1} + \sqrt{2t-1}}{\sqrt{2t+2h-1} + \sqrt{2t-1}}$$

(multiply by conjugate)

$$= \lim_{h \to 0} \frac{2t+2h-1 - (2t-1)}{h(\sqrt{2t+2h-1} + \sqrt{2t-1})}$$

$$= \lim_{h \to 0} \frac{\cancel{2t}+2h-\cancel{1}-\cancel{2t}+\cancel{1}}{h(\sqrt{2t+2h-1} + \sqrt{2t-1})}$$

$$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2t+2h-1} + \sqrt{2t-1})}$$

$$= \frac{2}{\sqrt{2t-1} + \sqrt{2t-1}}$$

$$= \frac{2}{2\sqrt{2t-1}} = \boxed{\frac{1}{\sqrt{2t-1}}}$$

3b) $g'(5) = \frac{1}{\sqrt{2\cdot 5 - 1}} = \frac{1}{\sqrt{9}} = \boxed{\frac{1}{3}}$

3c) $y = g(t) = \sqrt{2t-1}$

$g(5) = \sqrt{2\cdot 5 - 1} = \sqrt{9} = 3$

slope $m = g'(5) = \frac{1}{3}$, point $= (5,3)$

Point-slope Eq: $y = y_0 + m(x - x_0)$

$$\boxed{y = 3 + \frac{1}{3}(x-5)}$$

4) a) $f(x) = 6x^2 - 10x - 5x^{-2} + (2-x)^{-1} - 3(5-x^2)^{-1}$

$f'(x) = 12x - 10 + 10x^{-3} - (2-x)^{-2}(-1) + 3(5-x^2)^{-2}(-2x)$

$= \boxed{12x - 10 + \frac{10}{x^3} + \frac{1}{(2-x)^2} - \frac{6x}{(5-x^2)^2}}$ (power chain rule)

4b) $r(\theta) = \theta \sin\theta + \cos\theta$

$r'(\theta) = \theta\cos\theta + \sin\theta - \sin\theta$ (product rule)

$= \boxed{\theta\cos\theta}$

4c) $y = \sec\left(\frac{x}{2}\right)\tan(x^2)$

$y' = \sec\left(\frac{x}{2}\right)\left[\sec^2(x^2)(2x)\right] + \tan(x^2)\left[\sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)\right]$

(product rule & chain rule)

$= \boxed{2x\sec\left(\frac{x}{2}\right)\sec^2(x^2) + \frac{1}{2}\sec\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)\tan(x^2)}$

4d) $g(t) = (5-2t)^{-3} + \frac{1}{8}(2t^{-1}+1)^4$

$g'(t) = -3(5-2t)^{-4}(-2) + \frac{1}{2}(2t^{-1}+1)^3(-2t^{-2})$

$= \boxed{\frac{6}{(5-2t)^4} - \frac{\left(\frac{2}{t}+1\right)^3}{t^2}}$ (power chain rule)

4e) $y = 4\sin\left(\sqrt{1+\sqrt{x}}\right) = 4\sin\left((1+x^{1/2})^{1/2}\right)$

$y' = 4\cos\left(\sqrt{1+\sqrt{x}}\right)\left(\frac{1}{2}(1+x^{1/2})^{-1/2}\left(\frac{1}{2}x^{-1/2}\right)\right)$

$= 4\cos\left(\sqrt{1+\sqrt{x}}\right)\left(\frac{1}{4}\frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}\right)$

$= \boxed{\frac{\cos\left(\sqrt{1+\sqrt{x}}\right)}{\sqrt{x+x\sqrt{x}}}}$

5a) $\lim_{t\to 0} \frac{6\sin t}{t} = 6\lim_{t\to 0}\frac{\sin t}{t} = 6\cdot 1 = \boxed{6}$

5b) $\lim_{t\to 0}\frac{2t}{\tan t} = \lim_{t\to 0}\frac{2t}{\sin t/\cos t} = \lim_{t\to 0}\frac{2t\cos t}{\sin t}\cdot\frac{1/t}{1/t}$

$= \lim_{t\to 0}\frac{2\cos t}{(\sin t)/t} = \frac{\lim_{t\to 0} 2\cos t}{\lim_{t\to 0}(\sin t)/t} = \frac{2\cdot 1}{1} = \boxed{2}$

6a) $f(x)$ is discontinuous at $x = 2, 6, 8$.

6b) At $x=2$, $\lim_{x\to 2} f(x)$ does not exist.

At $x=6$, $\lim_{x\to 6} f(x) \neq f(6)$.

At $x=8$, $\lim_{x\to 8} f(x)$ does not exist.