1. (a) If function f is continuous on a closed interval $[a, b]$, then f has both an absolute maximum value and an absolute minimum value in $[a, b]$.

   (b) $y = 27x - x^3, \quad [-4, 2]$

   y is continuous on $[-4, 2]$ so absolute extrema exist.

Critical points:	$y' = 27 - 3x^2 = 0$ at $(-3, -54)$. y' is defined for every point in this interval.
Endpoints:	$(-4, 44), (2, 46)$
Absolute maximum:	$(2, 46)$
Absolute minimum:	$(-3, -54)$

2. (a) Mean Value Theorem: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one point c in (a, b) at which

   $$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

   (b) $y = f(x) = x^{2/3}, \quad [0, 8]$

   Because y is continuous on $[0, 8]$ and differentiable on $(0, 8)$, it satisfies the hypothesis of the Mean Value Theorem.

   $y' = f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

   There is a c in $(0, 8)$ at which

   $$\begin{aligned} f'(c) &= \frac{f(8) - f(0)}{8 - 0} \\ \frac{2}{3\sqrt[3]{c}} &= \frac{4 - 0}{8} = \frac{1}{2} \\ 3\sqrt[3]{c} &= 4 \\ \sqrt[3]{c} &= \frac{4}{3} \\ c &= \frac{64}{27}. \end{aligned}$$

3. (a) $y' = 0$ at $x = 1$. y' is undefined at $x = 0$.

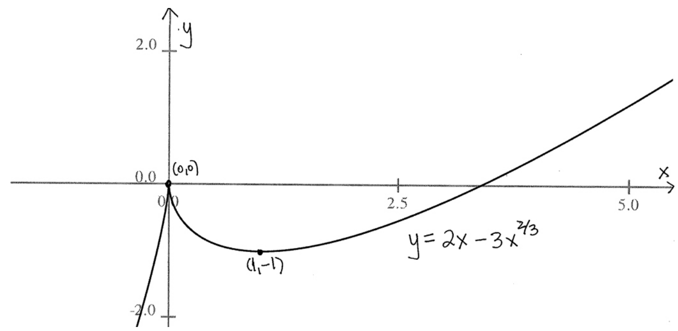
y’:	+ + +	|	− − −	|	+ + +
	incr	0	decr	1	incr

   (b) The critical points are $(0, 0)$ and $(1, -1)$. The point $(0, 0)$ is a local maximum and the point $(1, -1)$ is a local minimum. There are no absolute extrema.

   (c) y'' is undefined at $x = 0$.

y”:	+ + +	|	+ + +
	up	0	up

(d) There are no inflection points. The graph is concave up throughout.



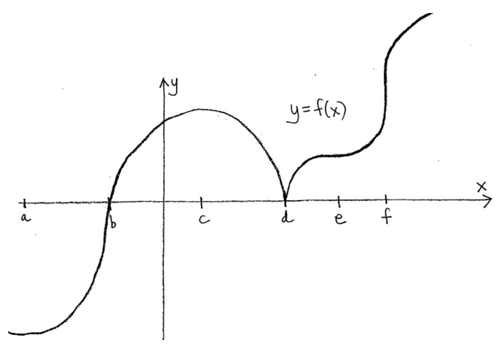
4. Let h represents the height of the grain and V represent the volume of the grain. We wish to find dh/dt when $h=12$ given $dV/dt=-30$, . We also know that $\frac{r}{h}=\frac{10}{20}=\frac{1}{2} \implies r=\frac{h}{2}$.

$$\begin{aligned} V &= \frac{\pi}{3}r^2h = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi}{12}h^3 \\ \frac{dV}{dt} &= \frac{\pi}{4}h^2\frac{dh}{dt} \\ -30 &= \frac{\pi}{4}(12^2)\frac{dh}{dt} \\ \frac{dh}{dt} &= -\frac{30}{36\pi} = -\frac{5}{6\pi}\text{ ft/min} \end{aligned}$$

5. (a) False. The function f may not have any extrema at the critical points of its first derivative. For example, $f(x)=x^3$ has a critical point at $x=0$, where $f'(x)=0$, but f has no maximum or minimum values there.

   (b) True. Inflection points for a function are found where the second derivative equals 0 or is undefined.

6. (a)



   (b) $y'=0$ at $x=a,c,e$. y' is undefined at $x=d,f$.
   $y''=0$ at $x=b,e$. y'' is undefined at $x=d,f$.

y':	$---$	|	$+++$	|	$---$	|	$+++$	|	$+++$	|	$+++$
	decr	a	incr	c	decr	d	incr	e	incr	f	incr

y'':	$+++$	|	$---$	|	$---$	|	$+++$	|	$---$
	up	b	down	d	down	e	up	f	down