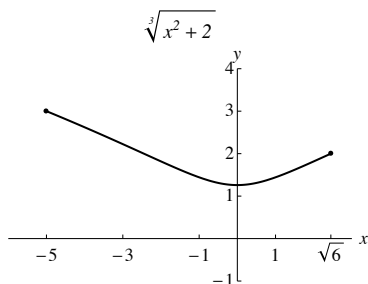


1. (a) $\lim_{\theta \rightarrow 0} \frac{3 \sin 5\theta}{4\theta} = \lim_{\theta \rightarrow 0} \frac{3}{4} \cdot \frac{\sin 5\theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{3}{4} \cdot \frac{5}{1} \cdot \frac{\sin 5\theta}{5\theta} = \frac{3}{4} \cdot \frac{5}{1} \cdot 1 = \boxed{\frac{15}{4}}$.

(b) $\lim_{\theta \rightarrow 0} \theta^2 \cot 3\theta \csc 2\theta = \lim_{\theta \rightarrow 0} \frac{\theta^2}{1} \cdot \frac{\cos 3\theta}{\sin 3\theta} \cdot \frac{1}{\sin 2\theta} = \lim_{\theta \rightarrow 0} \frac{\cos 3\theta}{1} \cdot \frac{\theta}{\sin 3\theta} \cdot \frac{\theta}{\sin 2\theta} = \lim_{\theta \rightarrow 0} \frac{\cos 3\theta}{1} \cdot \frac{1}{3} \cdot \frac{3\theta}{\sin 3\theta} \cdot \frac{1}{2} \cdot \frac{2\theta}{\sin 2\theta} = 1 \cdot \frac{1}{3} \cdot 1 \cdot \frac{1}{2} \cdot 1 = \boxed{\frac{1}{6}}$.

2.



$g(x) = \sqrt[3]{x^2 + 2}$ on $[-5, \sqrt{6}]$.

First we examine the endpoints.

At $x = -5$, $g(-5) = \sqrt[3]{(-5)^2 + 2} = \sqrt[3]{25 + 2} = \sqrt[3]{27} = 3$.

At $x = \sqrt{6}$, $g(\sqrt{6}) = \sqrt[3]{(\sqrt{6})^2 + 2} = \sqrt[3]{6 + 2} = \sqrt[3]{8} = 2$.

The endpoints are $(-5, 3)$ and $(\sqrt{6}, 2)$.

Next we examine the critical points where g' may be undefined or zero.

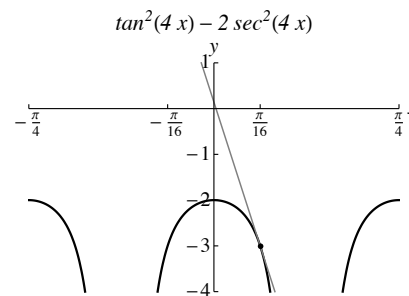
$$\begin{aligned} g(x) &= \sqrt[3]{x^2 + 2} \\ &= (x^2 + 2)^{1/3} \\ g'(x) &= \frac{1}{3}(x^2 + 2)^{-2/3}[x^2 + 2]' \\ &= \frac{1}{3}(x^2 + 2)^{-2/3}(2x) \\ &= \frac{2x}{3(x^2 + 2)^{2/3}} \end{aligned}$$

The derivative g' is defined for all x because $x^2 + 2$ is always positive.

The derivative equals 0 when $x = 0$. The corresponding y value is $g(0) = \sqrt[3]{0^2 + 2} = \sqrt[3]{2}$. The only critical point is $(0, \sqrt[3]{2})$.

Therefore the absolute maximum value is $\boxed{3}$ at $x = -5$ and the absolute minimum value is $\boxed{\sqrt[3]{2}}$ at $x = 0$.

3.



(a)

$$\begin{aligned} f(x) &= \tan^2 4x - 2 \sec^2 4x \\ f'(x) &= 2 \tan 4x [\tan 4x]' - 4 \sec 4x [\sec 4x]' \\ &= 2 \tan 4x \sec^2 4x [4x]' - 4 \sec 4x \sec 4x \tan 4x [4x]' \\ &= 2 \tan 4x \sec^2 4x (4) - 4 \sec 4x \sec 4x \tan 4x (4) \\ &= 8 \tan 4x \sec^2 4x - 16 \tan 4x \sec^2 4x \\ &= \boxed{-8 \tan 4x \sec^2 4x} \end{aligned}$$

Alternate Solution: Use the identity $\tan^2 \theta + 1 = \sec^2 \theta$.

$$\begin{aligned} f(x) &= \tan^2 4x - 2 \sec^2 4x \\ &= \tan^2 4x - 2(\tan^2 4x + 1) \\ &= -\tan^2 4x - 2 \\ f'(x) &= -2 \tan 4x [\tan 4x]' \\ &= -2 \tan 4x \sec^2 4x [4x]' \\ &= -2 \tan 4x \sec^2 4x (4) \\ &= \boxed{-8 \tan 4x \sec^2 4x} \end{aligned}$$

(b) The tangent slope at $x = \frac{\pi}{16}$ is

$$\begin{aligned} f' \left(\frac{\pi}{16} \right) &= -8 \tan \left(4 \cdot \frac{\pi}{16} \right) \sec^2 \left(4 \cdot \frac{\pi}{16} \right) \\ &= -8 \tan \left(\frac{\pi}{4} \right) \sec^2 \left(\frac{\pi}{4} \right) \\ &= -8 \cdot 1 \cdot \frac{1}{\cos^2 \left(\frac{\pi}{4} \right)} \\ &= -8 \cdot 1 \cdot \frac{1}{1/2} \\ &= -8 \cdot 1 \cdot 2 = -16. \end{aligned}$$

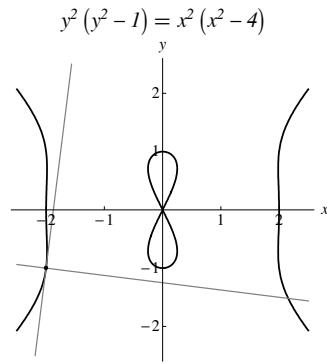
The function value at $x = \frac{\pi}{16}$ is

$$\begin{aligned} f \left(\frac{\pi}{16} \right) &= \tan^2 \left(4 \cdot \frac{\pi}{16} \right) - 2 \sec^2 \left(4 \cdot \frac{\pi}{16} \right) \\ &= \tan^2 \left(\frac{\pi}{4} \right) - 2 \sec^2 \left(\frac{\pi}{4} \right) \\ &= (1)^2 - \frac{2}{\cos^2 \left(\frac{\pi}{4} \right)} \\ &= (1)^2 - \frac{2}{1/2} \\ &= 1 - 4 = -3. \end{aligned}$$

Using the slope $m = -16$ and point $\left(\frac{\pi}{16}, -3 \right)$, the equation of the tangent line at $x = \frac{\pi}{16}$ is

$$\begin{aligned} y &= y_1 + m(x - x_1) \\ y &= -3 - 16 \left(x - \frac{\pi}{16} \right) \\ y &= -16x + \pi - 3. \end{aligned}$$

4.



(a)

$$\begin{aligned} y^2(y^2 - 1) &= x^2(x^2 - 4) \\ y^4 - y^2 &= x^4 - 4x^2 \\ 4y^3 \frac{dy}{dx} - 2y \frac{dy}{dx} &= 4x^3 - 8x \\ \frac{dy}{dx} (4y^3 - 2y) &= 4x^3 - 8x \\ \frac{dy}{dx} &= \frac{4x^3 - 8x}{4y^3 - 2y} = \boxed{\frac{2x^3 - 4x}{2y^3 - y}}. \end{aligned}$$

(b) The tangent slope is

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{(-2, -1)} &= \frac{2(-2)^3 - 4(-2)}{2(-1)^3 - (-1)} \\ &= \frac{-16 + 8}{-2 + 1} = \frac{-8}{-1} = 8. \end{aligned}$$

The normal slope, which equals the negative reciprocal of the tangent slope, is $-1/8$. An equation for the normal line with slope $m = -1/8$ and point $(-2, -1)$ is

$$\begin{aligned} y &= y_1 + m(x - x_1) \\ y &= -1 - \frac{1}{8}(x + 2) \\ y &= -\frac{1}{8}x - \frac{5}{4} \end{aligned}$$

5. (a) The function f must be continuous on $[a, b]$ and differentiable on (a, b) to satisfy the hypotheses of the Mean Value Theorem. Since f is a polynomial function, it is continuous and differentiable for all x .

(b) We differentiate the function to get

$$\begin{aligned} f'(x) &= 6x \\ f'(c) &= 6c \end{aligned}$$

and substitute into the formula

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$6c = \frac{3b^2 - 3a^2}{b - a}$$

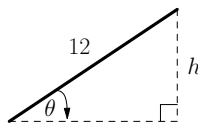
$$6c = \frac{3(b^2 - a^2)}{b - a}$$

$$6c = \frac{3(b + a)(b - a)}{b - a}$$

$$6c = 3(b + a)$$

$$c = \frac{a + b}{2}$$

6.



(a) Since $\sin \theta = h/12$ and $\sin(\frac{\pi}{6}) = 1/2$, then

$$\frac{h}{12} = \frac{1}{2}$$

$$h = 6.$$

When closed the bridge stands 3 meters above the water so the center of the bridge is $6 + 3 = 9$ meters above the water when $\theta = \frac{\pi}{6}$.

(b) Given: $d\theta/dt = -\frac{\pi}{32}$. In the previous problem we found that when $\theta = \frac{\pi}{6}$, then $h = 6$.

We differentiate the sine function:

$$\sin \theta = \frac{h}{12}$$

$$\cos \theta \frac{d\theta}{dt} = \frac{1}{12} \cdot \frac{dh}{dt}$$

$$\left(\cos \frac{\pi}{6}\right) \cdot \left(-\frac{\pi}{32}\right) = \frac{1}{12} \cdot \frac{dh}{dt}$$

$$\frac{\sqrt{3}}{2} \cdot \left(-\frac{\pi}{32}\right) = \frac{1}{12} \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = -12 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\pi}{32}$$

$$= \boxed{-\frac{3\sqrt{3}\pi}{16} \text{ m/sec}}$$

(c) (This question can be answered without part (b).)

The bridge is closing at the constant rate of $\frac{\pi}{32}$ radians per second. (Note that we cannot use dh/dt because it is not a constant rate.) Starting when $\theta = \frac{\pi}{6}$, the number of seconds before the bridge is completely closed is

$$\frac{\frac{\pi}{6}}{\frac{\pi}{32}} = \frac{32}{6} = \frac{16}{3} = 5\frac{1}{3} \text{ seconds.}$$

Bond is 170 meters away and traveling at the constant speed of 30 m/sec. The number of seconds before he gets to the bridge is

$$\frac{170}{30} = \frac{17}{3} = 5\frac{2}{3} \text{ seconds.}$$

No, Bond will not get there in time. The bridge will be closed when he arrives. He will need to leap over the bridge or dive under it.

Extra Credit

Let s represent the distance from any point (x, y) to the origin. We are given that $dx/dt = 3$. We wish to find ds/dt when $x = 1$.

We use the distance formula $s = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. The distance from any point (x, y) to the origin is $s = \sqrt{x^2 + y^2}$.

When $x = 1$, $y = (1 + 1)^{3/2} = 2\sqrt{2}$ and $s = \sqrt{x^2 + y^2} = \sqrt{1^2 + (2\sqrt{2})^2} = 3$.

Next substitute $y = (x + 1)^{3/2}$ into the distance formula.

$$\begin{aligned}s &= \sqrt{x^2 + y^2} \\s^2 &= x^2 + y^2 \\s^2 &= x^2 + ((x + 1)^{3/2})^2 \\s^2 &= x^2 + (x + 1)^3\end{aligned}$$

Then differentiate and solve.

$$\begin{aligned}s^2 &= x^2 + (x + 1)^3 \\2s \frac{ds}{dt} &= 2x \frac{dx}{dt} + 3(x + 1)^2 \frac{dx}{dt} \\2s \frac{ds}{dt} &= (2x + 3(x + 1)^2) \frac{dx}{dt} \\2(3) \frac{ds}{dt} &= (2(1) + 3(1 + 1)^2)(3) \\6 \frac{ds}{dt} &= (2 + 12)(3) \\ \frac{ds}{dt} &= \frac{42}{6} = \boxed{7 \text{ units/sec}}.\end{aligned}$$