APPM 1340 **Exam 3** **Fall 2009**

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) section number, and (3) a grading table on the front of your bluebook. **Start each problem on a new page. Simplify your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Unless otherwise indicated, **show all work**.

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1. (10 points) Evaluate the following limits.

   (a) $\lim_{\theta \to 0} \dfrac{3 \sin 5\theta}{4\theta} =$

   (b) $\lim_{\theta \to 0} \theta^2 \cot 3\theta \csc 2\theta =$

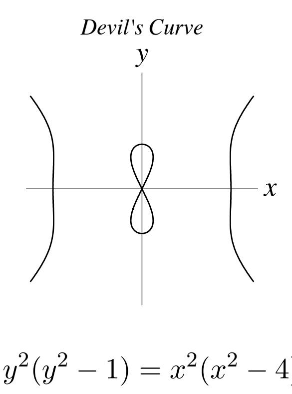
2. (15 points) Find all absolute maximum and minimum values for $g(x) = \sqrt[3]{x^2 + 2}$ on $\left[-5, \sqrt{6}\right]$.

3. (20 points) Let
$$f(x) = \tan^2 4x - 2\sec^2 4x.$$

   (a) Find $f'(x)$. Simplify your answer.

   (b) Find an equation for the line tangent to the curve at $x = \frac{\pi}{16}$.

4. (20 points)



$$y^2(y^2 - 1) = x^2(x^2 - 4)$$

   (a) Use implicit differentiation to find dy/dx. You need not simplify.

   (b) Find an equation for the line normal to the curve at $(-2, -1)$.

5. (10 points) Let $f(x) = 3x^2$ on $[a, b]$ for constants a and b.

   (a) Show that f satisfies the hypotheses of the Mean Value Theorem.

   (b) Show that the value of c in the conclusion of the Mean Value Theorem is $(a + b)/2$, the arithmetic mean of a and b.

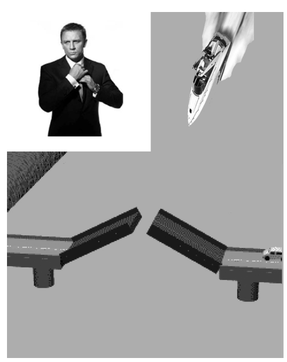
6. (25 points) James Bond is racing his 5-meter-high speedboat at 30 m/sec toward an open drawbridge which has begun to close, trying to pass through before the bridge returns to the horizontal position. When closed the bridge spans 24 meters (in two 12-meter sections) and stands 3 meters above the water.

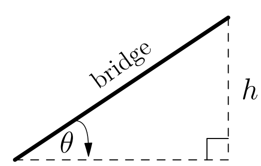
   Let θ represent the angle the open bridge forms with the horizontal, and h represent the height of the center of the bridge above the resting position. The angle θ is decreasing at the constant rate of $\frac{\pi}{32}$ radians per second.

   (a) How high above the water is the center of the bridge when $\theta = \frac{\pi}{6}$ radians?

   (b) How fast is the height h decreasing then?

   (c) At that moment the speedboat is 170 meters away. If Bond maintains his speed, will the boat pass through safely in time? Explain.





Extra Credit (10 points)

A tiny bug moves along the curve $y = (x+1)^{3/2}$ in the first quadrant so that its x-coordinate increases at the rate of 3 units per second. Find the rate its distance from the origin is changing when $x = 1$.