Exam #3: APPM 1350 - Spring 2005.

ON THE FRONT OF YOUR BLUEBOOK please write: (1) your name, (2) your student ID, (3) your section and lecturer name (010-Carvalho or 020-Lladser). You must work all the problems on the exam. Show all your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, and calculators are not permitted.

P1. (25 points) Determine whether the following statements are TRUE (i.e. always true), FALSE (i.e. always false) or NEITHER (i.e. sometimes true and sometimes false):

(a) If the linearization of $f(x)$ at $x = 1$ is $L(x) = 2$ then $f'(1) = 0$.

(b) If Newton's method is used to find a solution of the equation $x^3 + x^2 - x - 1 = 0$ starting at the point $x_0 = 2$ then $x_1 = 1$.

(c) If $\int f(x)\,dx = \int g(x)\,dx$, for all $x \in \mathbb{R}$, then $f(x) = g(x)$, for all $x \in \mathbb{R}$.

(d) If $f'(x) = \tan^3(x)$, for all $-\pi/2 < x < \pi/2$, then $\int f(x)\,dx = \tan^3(x) + C$, for some constant C.

(e) $\frac{d}{dx}\left[\int_0^1 \frac{1+t^2}{1+t^3}\,dt\right] = \frac{1+x^2}{1+x^3}$, for all $0 < x < 1$.

P2. (30 points) Evaluate the following integrals. Simplify your answer to all definite integrals.

(a) $\int (w^{1/3} - 2) \cdot (w^{2/3} + 1)\,dw$

(b) $\int \sec^2(x)\sqrt{1 + \sqrt{\tan(x)}}\,dx$

(c) $\int_{1/2}^{1} \frac{1}{y^2} - \frac{1}{y^3}\,dy$

(d) $\int_0^{\pi} \sin^2\left(\frac{z}{2}\right)\cos\left(\frac{z}{2}\right)\,dz$

P3. (20 points) A Recreation Center would like to build a circular jacuzzi (i.e. the shape is a right circular cylinder) to hold 1000 cubic feet of water. If the cost of the tile flooring is \$1 per square foot and the cost of the tile for the side is \$2 per square foot, find the optimum dimensions (i.e. radius and depth) of the jacuzzi that will minimize the total cost of the tile. Justify that you have indeed found the minimum.

(ONE MORE PROBLEM ON THE BACK)

P4. (25 points) Let $f : (-\pi/2, \pi/2) \to \mathbb{R}$ be a continuous function, $a \in (-\pi/2, \pi/2)$ be a given number and let $y(x)$, with $x \in (-\pi/2, \pi/2)$, be the function defined as:

$$y(x) = \sec(x) \cdot \int_a^x f(t) \cdot \cos(t)\, dt\,.$$

(a) Compute $\dfrac{dy}{dx}$. Justify your steps and simplify your answer.

(b) Use part (a) to show that $y(x)$ is a solution for the following differential equation:

$$\frac{dy}{dx} - \tan(x) \cdot y = f(x).$$

(c) Compute $y(a)$.