Here are some conceptual problems for you to contemplate as you prepare for the first exam.

1. In words, explain the meaning of absolute value in the equation $|x-3|=7$.
   (a) On the number line show what values of x would satisfy $|x-3|=7$.
   (b) Explain what $|x-3|<7$ means in terms of distance. (That is, the distance from the number x to 3 is less than 7). Illustrate this on a number line.
   (c) On the number line, show which values of x would satisfy $|x-3|\geq 7$.
2. What numbers would be solutions to $|x|+|y|\geq 4$. Illustrate this on a graph.
3. Explain how you would prove that a triangle with vertices located at $(0,0)$, $(1,5)$ and $(2,0)$ is an isosceles triangle.
4. Graph the following functions using what you know about translations.
   (a) $f(x)=-\frac{1}{2}(x-4)^2+1$ (b) $f(x)=-2\sin\left(\frac{x}{2}\right)-3$
5. Suppose you wanted to graph the equation $x^2+y^2-4x-5=0$. How would you get it into standard form? Draw its graph.
6. Suppose you know that f(x) is an even function and $\lim_{x\to 7^-} f(x)=2$.
   (a) What can you say about $\lim_{x\to -7^-} f(x)=2$?
   (b) Draw a graph to illustrate your point.
   (c) Now, suppose the function were odd?
7. Why is the period of $y=\sin(4x)$ shorter than $y=\sin(x)$?
8. For each of the following problems, first try to find the limits by substitution, and explain what you would do after substitution.
   (a) $\lim_{x\to 4}\frac{\sqrt{x-3}}{x-4}$ (c) $\lim_{x\to 3}\frac{x^2+9}{x+3}$
   (b) $\lim_{x\to 4}\frac{x^2-16}{x-4}$ (d) $\lim_{h\to 0}\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}$
9. Find the derivatives of the following functions from first principles.
   (a) $y=\frac{1}{\sqrt{x+3}}$ (b) $y=x^2+6x$
10. What does it mean if I say $\lim_{x\to 7} f(x)=2$?

11. If you know that $\lim_{x \to 7} f(x)$ exists, can you find its value by calculating $\lim_{x \to 7^-} f(x)$. Explain.

12. Suppose you know that a function is continuous at $x = a$. Which of the following are true?

    (a) The function is differentiable at $x = a$

    (b) The function has a limit at $x = a$

    (c) The function has a left and right hand limit at $x = a$

    (d) The function has a defined value at $x = a$

    (e) All of the above

13. How could I use the IVT (intermediate value theorem) to help me find a root of an equation? (Hint: show that the function $f(x) = x^3 - 15x + 1 = 0$ has at least one solution in the interval $[-4, 4]$).

14. Draw four graphs of functions for which the derivative fails to exist at a point. (For each graph, the derivative should fail to exist for a different reason.)

15. Can you use the product rule to find the derivative of the function $y = -3x$? If so, what happens?

16. What would the derivative of $y(x) = u(x)\, v(x)\, w(x)$ be? Prove it.