

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.

- (a) If $f(x)$ is continuous at x_0 , then it is differentiable at x_0 .
 (b) If $|2x - 3| > 1$, then $1 < x < 2$.
 (c) If $f(x)$ is an odd function and $\lim_{x \rightarrow a^+} f(x) = L$, then $\lim_{x \rightarrow -a^-} f(x) = -L$
 (d) If $p(x)$ is a polynomial, then $\lim_{x \rightarrow 2^+} \frac{p(x)}{x - 2} = +\infty$.
 (e) If $f'(x) = g'(x)$, then $f(x) = g(x)$.

2. (25 points) Compute the following limits, if they exist. If the limit does not exist, clearly state this.

- (a) $\lim_{x \rightarrow 0} x^4 \sin\left(\frac{1}{x}\right)$ (d) $\lim_{x \rightarrow 4} \frac{\frac{1}{x} - \frac{1}{4}}{x - 4}$
 (b) $\lim_{x \rightarrow -1} \frac{x^2 - 4x - 5}{x^2 - 1}$ (e) $\lim_{x \rightarrow 3} f(x)$ where
 (c) $\lim_{x \rightarrow 2^-} \frac{x - 3}{x^2 - 4}$ $f(x) = \begin{cases} 2x^2 - 6x + 5, & \text{if } x \leq 3 \\ x^3 - x^2 - 3x, & \text{if } x > 3 \end{cases}$

3. (25 points) Find the requested information.

- (a) State the definition of the derivative, $\frac{df}{dx}$.
 (b) Using this definition, calculate the derivative with respect to x of $f(x) = x^2 + x$.
 (c) Determine the equation of the tangent line to $f(x)$ at $x = 2$.
 (d) Determine the equation of the normal line to $f(x)$ at $x = 2$.

4. (20 points) Using appropriate rules of differentiation, evaluate the following derivatives. For parts (a) and (b) do not simplify your work beyond the point of having no derivatives remaining in your result.

- (a) $\frac{d}{dx} \left(3x^4 + \frac{7}{x^2} + \pi^2 \right)$ (c) $f'(2)$ if $f(x) = \frac{u(x)v(x)}{w(x)}$ where
 (b) $f'(x)$ where $f(x) = \frac{x^2 + 3}{x(x^4 + 1)}$ $u(2) = 3, v(2) = 4, w(2) = 2$, and
 $u'(2) = 1, v'(2) = 5$ and $w'(2) = 5$

5. (15 points) Are there numbers a and b that will make the function

$$f(x) = \begin{cases} ax^2 + 3b & \text{for } x < 1 \\ ax^4 - 3x + 4b & \text{for } x \geq 1 \end{cases}$$

differentiable at $x = 1$? If yes, determine their values. Be sure to show your work and clearly justify your answers.