

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and **box** your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.

- (a) The derivative of a differentiable even function is an odd function.
 (b) The point $x = 0$ is an inflection point of the function $y = x^4 - 2x + 6$.
 (c) If $f(x)$ is differentiable everywhere and $f(3) = f(6) = 0$, then $f(x)$ has a critical point in $(3, 6)$.
 (d) If $f(x)$ and $g(x)$ are increasing functions on an interval I , then the product $f(x)g(x)$ is increasing on I .
 (e) The graph of $y = \frac{4x^{2005} + 30x^{1776} - x}{7x^{2005} + 5x^{1215}}$ has a horizontal asymptote.

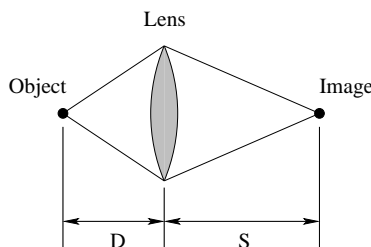
2. (30 points) Compute the following. (If a limit does not exist, clearly state this.)

- (a) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\tan(3x)}$ (c) $\lim_{x \rightarrow \infty} \frac{x^{-1} + 3x^{-2}}{4x^{-3} + 8x^{-1}}$
 (b) $\lim_{x \rightarrow \infty} \frac{x^2 + 3x - \sin x}{4 + 3x^2 + \cos(x^2)}$ (d) $\frac{dy}{dx}$ if $\sec^2(xy) = x^2 + y^3$
 (e) $\frac{dy}{dx}$ if $y = (2x - 3)^5(x^2 + 5)^7$

3. (20 points) The thin lens equation in optics is

$$\frac{1}{D} + \frac{1}{S} = \frac{1}{F},$$

where D is the distance from the object to the lens, S is the distance from the image to the lens, and F , a constant, is the focal length of the lens. Suppose that a particular lens has a focal length of $F = 6$ cm and that an object is moving towards the lens at the rate of 2 cm/s. How fast is the image distance S changing at the moment when the distance between the object and the lens is $D = 10$ cm?



4. (20 points) You want to build a pool in your backyard for your prize-winning Koi. The pool should have a square bottom and hold V gallons of water. You want to put decorative tile on the vertical sides of the pool, but to save money, the bottom will be plain concrete. The tile costs four times as much per square foot as the concrete. What pool dimensions will minimize the cost?

THERE IS MORE ON THE BACK

5. (15 points) A twice-differentiable function $y = f(x)$ has an x -intercept at $x = 4$. In addition, the signs of the first and second derivatives in the given intervals are as follows:

	$(-\infty, 0)$	$(0, 2)$	$(2, 4)$	$(4, 6)$	$(6, \infty)$
$f'(x)$	+	-	-	+	+
$f''(x)$	-	-	+	+	-

Using all of the above information, carefully sketch the graph of $y = f(x)$, labeling all maxima, minima and inflection points.