APPM 1350 **EXAM 3** **FALL 2005**

INSTRUCTIONS: Books, crib sheets and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation number on the front of your bluebook. Work all problems. Start each problem on a **new page**. Show your work clearly and box your final answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

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1. (20 points) Answer the following questions as either **ALWAYS TRUE** or **NOT ALWAYS TRUE**. For this problem only, you do not need to justify your answer.

   (a) If $f(x) = x^n$, where n is an odd integer, then $f^{-1}(x)$ exists for all values of x.

   (b) If $\int_0^2 f(x)\,dx = M$ and $\int_0^2 g(x)\,dx = N$, then $\int_0^2 f(x) \cdot g(x)\,dx = M \cdot N$.

   (c) The area between an integrable function $f(x)$ and the x–axis for $a \le x \le b$ is given by $\int_a^b f(x)\,dx$.

   (d) The trapezoidal rule overestimates a definite integral if the integrand is positive and concave up over the interval of integration.

   (e) If $f(x) = f(-x)$ over the interval $[-a, a]$, then $\int_{-a}^a f(x)\,dx = 0$.

2. (30 points) Compute the following.

   (a) $\dfrac{d}{dx}\displaystyle\int_3^{x^3} \sqrt{t^2 + t^4}\,dt$

   (b) $\displaystyle\int x^{1/3} \sin(x^{4/3} - 8)\,dx$

   (c) $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}\,\sin^2(\sqrt{x})}\,dx$

   (d) $\displaystyle\int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2+1}}\,dx$

   (e) $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \frac{4x}{\sqrt{x^{10}+1}}\,dx$

3. (20 points) As we have recently seen in lecture, the natural log function is defined as $\ln(x) = \displaystyle\int_1^x \frac{dt}{t}$. Suppose we want to estimate the value of $\ln(4) = \displaystyle\int_1^4 \frac{dt}{t}$ by using numerical integration.

   (a) First, write out the general trapezoidal rule to approximate the value of $\displaystyle\int_a^b f(x)\,dx$.

   (b) Use the trapezoidal rule with $n = 3$ to estimate the value of $\ln(4)$.

   (c) When using the trapezoidal rule to estimate the value of $\ln(4)$, what is the smallest number of subintervals required to guarantee that the error is smaller than $\dfrac{1}{18}$? (Hint: $|E_T| \le \dfrac{b-a}{12}h^2M$.)

4. (15 points) Again, considering the natural log function defined as $\ln(x) = \displaystyle\int_1^x \frac{dt}{t}$, suppose we want to estimate the value of $\ln(1.2)$ by using a linearization of $\ln(x)$ near $x = 1$.

   (a) Write out the linearization of $\ln(x)$ near $x = 1$.

   (b) Now, use your linearization from part (a) to estimate the value of $\ln(1.2)$.

5. (15 points)

   (a) State the Mean Value Theorem (for definite integrals).

   (b) Calculate the mean value of the function $f(x) = x^2$ over the interval $[0, 2]$.

   (c) Determine the value of "c" described by the MVT.