1 a) TRUE

b) TRUE

c) FALSE $\lim_{x\to 1^-} f(x) = 1$ $\lim_{x\to 1^+} f(x) = 0$ $\lim_{x\to 1} f(x)$ does not exist

d) FALSE $\lim_{x\to 0} f(x) = 0$ but $f(0) = 1$ $\Rightarrow$ $f(x)$ is discontinuous at $x=0$

e) TRUE

f) TRUE

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2 a) $f(x)$ is continuous at $x = x_0$ if $\boxed{\lim_{x\to x_0} f(x) = f(x_0)}$

b) $\lim_{x\to 3^-} f(x) = 3^2 - 1 = 8$ $\lim_{x\to 3^+} f(x) = (2a)3 = 6a$ & $f(3) = 6a$

So f will be continuous ($\lim_{x\to 3} f(x)$ exists & $= f(3)$) if $6a = 8$

$\Rightarrow \boxed{a = \frac{4}{3}}$

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3 a) As $x \to 0$ $\sqrt{x+1} \to 1$ & $x^2 \to 0^+$ $\Rightarrow$ $\lim_{x\to 0} \frac{\sqrt{x+1}}{x^2} = \boxed{\infty}$

(ie $\lim_{x\to 0^+} = \lim_{x\to 0^-} = \infty$)

b) Since $|\sin(\frac{1}{x})| \le 1$ $\forall x$, $-\sqrt{x} \le \sqrt{x}\sin(\frac{1}{x}) \le \sqrt{x}$

$\lim_{x\to 0^+} \pm\sqrt{x} = 0$ so, by the Sandwich Theorem, $\boxed{\lim_{x\to 0^+} \sqrt{x}\sin(\frac{1}{x}) = 0}$

c) $\lim_{x\to 6} \frac{1/x - 1/6}{x-6} = \lim_{x\to 6} \frac{\left(\frac{6-x}{6x}\right)}{x-6} = \lim_{x\to 6} \frac{-1}{6x} = \boxed{-\frac{1}{36}}$

d) $\frac{t}{|t|} = \begin{cases} 1 & t>0 \\ -1 & t<0 \end{cases}$ so $\lim_{t\to -2} \frac{t}{|t|} = \lim_{t\to -2} -1 = \boxed{-1}$

4 a) $\boxed{f'(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}}$ if this limit exists

b) $f'(1) = \lim_{h\to 0} \frac{1}{h}\left[2\sqrt{3(1+h)} - 2\sqrt{3}\right] = \lim_{h\to 0} \frac{2}{h} \frac{(\sqrt{3+3h} - \sqrt{3})(\sqrt{3+3h}+\sqrt{3})}{(\sqrt{3+3h}+\sqrt{3})}$

$= \lim_{h\to 0} \frac{2}{h} \cdot \frac{(3+3h) - 3}{\sqrt{3+3h}+\sqrt{3}} = \lim_{h\to 0} \frac{6\cancel{h}}{\cancel{h}(\sqrt{3+3h}+\sqrt{3})} = \frac{6}{\sqrt{3}+\sqrt{3}} = \frac{6}{2\sqrt{3}} = \boxed{\sqrt{3}}$

c) Tangent line has equation $y = mx + c$ where $m = f'(1) = \sqrt{3}$.
Line goes through $(x,y) = (1, 2\sqrt{3})$ so $2\sqrt{3} = \sqrt{3} + c \Rightarrow c = \sqrt{3}$.
$\Rightarrow$ eqn of tangent line is $\boxed{y = \sqrt{3}(x+1)}$

5 a) $y' = (x + \frac{1}{x})'(x - \frac{1}{x} + 1) + (x + \frac{1}{x})(x - \frac{1}{x} + 1)'$ (product rule)
$= \boxed{(1 - \frac{1}{x^2})(x - \frac{1}{x} + 1) + (x + \frac{1}{x})(1 + \frac{1}{x^2})} = 1 + 2x - \frac{1}{x^2} + \frac{2}{x^3}$

b) $y = \frac{x^4}{2} - \frac{3}{2}x^2 - x \Rightarrow y' = 2x^3 - 3x - 1$
$\Rightarrow y'' = 6x^2 - 3$
$\Rightarrow y^{(3)} = 12x \Rightarrow y^{(4)} = 12 \Rightarrow y^{(n)} = 0 \ \forall n \geq 5$

c) $g(2) = \frac{-2^2 + 4}{2+1} = 0$ $g'(x) = \frac{(-2x+2)(x+1) - (2x - x^2)(1)}{(x+1)^2} \Rightarrow g'(2) = \frac{-6-0}{9}$
$\Rightarrow g(2) + g'(2) = \boxed{-2/3}$

6. $f(0) = 10$ $f(-10) = 90 - 1000 = -910$. Since $f(x)$ is continuous and $f(-10) < -\sqrt{3} < f(0)$, by the INTERMEDIATE VALUE THEOREM, $f(c) = -\sqrt{3}$ for some $c \in (-10, 0)$

7. $\sin(x)$ is 2π-periodic $\Rightarrow \sin(2x)$ is π-periodic (ie $\boxed{\text{period} = \pi}$)