

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) instructor's name, (3) recitation section, and (4) "Exam 2" on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. This exam has 6 problems, on both sides of this sheet. Work all **6 problems**. Start each problem on a **new page**. Show your work. Box in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points)

- (a) Evaluate: $\lim_{x \rightarrow \infty} \frac{2x^3 + 7}{x^3 - x^2 + x + 7}$.
- (b) Calculate $\frac{dy}{dt}$ if $y = \sqrt{1 + \cos(t^2)}$.
- (c) Calculate $\frac{d}{d\theta} \sin\left(\frac{\pi}{3}\right)$.

2. (10 points)

(a) State the Mean Value Theorem. Be sure to include any conditions required for the theorem to apply.

(b) Let $f(x) = x \cdot \sqrt[3]{x} = x^{\frac{4}{3}}$, on $-1 \leq x \leq 8$. Does the Mean Value Theorem apply? Justify your answer.

3. (20 points) Let $x^3 + y^3 = 16$.

- (a) Find $\frac{dy}{dx}$ in terms of (x, y) .
- (b) Find $\frac{d^2y}{dx^2}$ in terms of $(x, y, \frac{dy}{dx})$.
- (c) Evaluate both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $\{x = 2, y = 2\}$. Be sure to say which is which.
- (d) Find the equation of the line tangent to the curve defined by $x^3 + y^3 = 16$ at $\{x = 2, y = 2\}$.

4. (20 points) Let $f(x) = 2x^4 - 4x^2 + 1$.

Sketch the graph of $y = f(x)$. Be sure to label all local maxima, minima, inflection points and cusps, and to give the coordinates of each. Also identify any asymptotes (horizontal, vertical or oblique) that exist for this function.

THERE IS MORE ON THE BACK

5. (15 points) Let $f(x) = 2x^4 - 4x^2 + 1$.

(a) Take one step in Newton's method to approximate the solution of $f(x) = 0$. Start at $x_0 = 2$. Find x_1 explicitly.

(b) Copy the graph you drew for problem 4 here. Show on your graph how Newton's method takes you from x_0 to x_1 , and from x_1 to x_2 (even though you did not calculate x_2).

[If your graph from problem 4 happens to be wrong, you will not lose more points here for a wrong graph. But your graphical depiction of Newton's method in (b) should be consistent with your calculation in (a).]

6. (25 points)

A window is in the form of a rectangle surmounted by a semi-circle. The rectangle is of clear glass, while the semicircle is of tinted glass that transmits only half as much light per unit area as the clear glass does. The total perimeter is fixed. Find the proportions of the window that admit the most light. Neglect the thickness of the frame. Be sure to include all the relevant lengths in your answer.

Did you write Exam 2 on the front of you bluebook?

Now that you have finished the exam, please decide whether to turn it in or to walk away with it. If you turn your exam in, your score on this exam will replace your previous score on exam 2. If you do not turn it in, your current score on exam 2 will stand.