

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) recitation section on the front of your bluebook. Also make a scoring table, with places for 5 problems, plus a total score. This exam has 5 problems, on both sides of this sheet. Work all **5 problems**. Start each problem on a **new page**. Show your work. Box in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive parital credit.

1. (9 points) For each statement below, answer TRUE if the statement is always true. If the statement is not always true, answer FALSE and give a counterexample showing why it is false.
 - a) If $\int f(x)dx = F(x) + C_1$ and $\int g(x)dx = G(x) + C_2$, then $\int f(x)g(x)dx = F(x)G(x) + C_3$
 - b) If $f(x) > 0$ and $f''(x) < 0$ for all real numbers, and if $a < b$, then the Trapezoidal Rule yields a number less than the value of $\int_a^b f(x)dx$ for $n < \infty$.
 - c) If $f(x)$ is a continuous function on a closed interval $[a,b]$ then there is a point c in $[a,b]$ such that $f(c)$ is the average value of $f(x)$ on the interval $[a,b]$.

2. (24 points) Consider the function $y = x^2 + 1$ and a partition of the interval $[-1,1]$ into four subintervals of equal length.
 - a) Draw a graph of the function and the Riemann sum rectangles, and using the right endpoint of each subinterval. Estimate the value of the $\int_{-1}^1 (x^2 + 1)dx$ using Riemann sums.
 - b) Draw a graph of the function and the trapezoids necessary to use Trapezoidal Rule, and estimate the value of the $\int_{-1}^1 (x^2 + 1)dx$ using Trapezoidal Rule. Note: $T = \frac{h}{2}(y_0 + 2[y_1 + y_2 + \dots + y_{n-1}] + y_n)$
 - c) What is the smallest value of n that would assure that if you use the trapezoidal rule to estimate $\int_{-1}^1 (x^2 + 1)dx$, you would have an error less than 10^{-3} , given that $|E_T| \leq \frac{b-a}{12} h^2 M$

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3. (16 points)

a) Given the function $f(x) = x^2 - 4x + 4$, $x > 2$, find the formula for f^{-1} .

b) Find the derivative of the following function: $y = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}$

4. (30 points)

a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos 3x}{(1+4\sin 3x)} dx$

b) Find the linearization of $y = 3x + \int_4^{x^2} \frac{\tan(t-4)}{t} dt$ near $x = -2$

c) Evaluate: $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}} dx$

5. (14 points)

a) Sketch the graph of $y = |x| - 1$

b) Calculate the average value of the function, $y = |x| - 1$, on the interval $[-1, 3]$.

6. (7 points) Consider $\int_a^b (3x - x^2) dx$ where $a < b$. What values should you assign “ a ” and “ b ” that would maximize the value of the integral?

Did you write Exam 3 on the front of your bluebook?

Now that you have finished the exam, please decide whether to turn it in or to walk away with it. If you turn your exam in, your score on this exam will replace your previous score on exam 3. If you do not turn it in, your current score on exam 3 will stand.