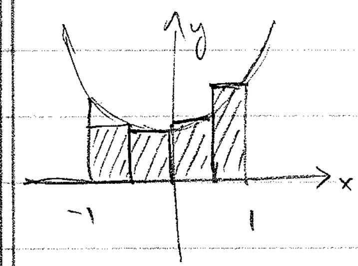
1a) FALSE $\int 2x\,dx = x^2 + C$ but $\int (2x)(2x)\,dx = \int 4x^2\,dx$

& $\int 4x^2\,dx = \frac{4}{3}x^3 + C \neq (x^2)(x^2) + C = x^4 + C$

b) TRUE

c) TRUE

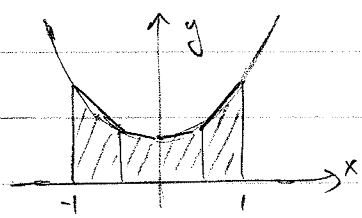
2a)



$$\int_{-1}^{1} x^2+1\,dx = \frac{1}{2}\left[\left(\left(-\tfrac{1}{2}\right)^2+1\right)+\left(0^2+1\right)+\left(\left(\tfrac{1}{2}\right)^2+1\right)+\left(1^2+1\right)\right]$$

$$= \frac{1}{2}\left[5+\frac{1}{4}+\frac{1}{4}\right] = \boxed{\frac{11}{4}}$$

b)



$$\int_{-1}^{1} x^2+1\,dx = \frac{1}{4}\left[\left((-1)^2+1\right)+\left((1)^2+1\right)+2\left[\left(\left(-\tfrac{1}{2}\right)^2+1\right)+\left(0^2+1\right)+\left(\left(\tfrac{1}{2}\right)^2+1\right)\right]\right]$$

$$= \frac{1}{4}\left[4+2\left[3+\frac{1}{2}\right]\right] = \boxed{\frac{11}{4}}$$

c) $|E_T| \le \frac{2}{12}h^2 M$ where $M \geq (x^2+1)'' = (2x)' = 2 \Rightarrow$ take $M = 2$.

$\Rightarrow |E_T| \le \frac{4}{12}h^2 = \frac{h^2}{3}$ Take h such that $|E_T| \le \frac{h^2}{3} \le 10^{-3} \Rightarrow h^2 \le 3\times10^{-3}$

$\Rightarrow h \le \sqrt{3\times10^{-3}}$

Since $h = \frac{2}{N} \Rightarrow N = \frac{2}{h}$ so take $N \geq \frac{2}{\sqrt{3\times10^{-3}}} \Rightarrow \boxed{N \geq \frac{200}{\sqrt{30}}}$

3a) $y = (x-2)^2 \Rightarrow \pm\sqrt{y} = x-2$ but $x > 2 \Rightarrow$ take $+\sqrt{y} = x-2 \Rightarrow x = \sqrt{y}+2$

$\Rightarrow \boxed{f^{-1}(x) = \sqrt{x}+2}$

b) $y = \frac{(x+1)^5}{(2x+1)^{5/2}}$ $y' = \frac{5(x+1)^4(2x+1)^{5/2} - \frac{5}{2}(2x+1)^{3/2}\cdot 2(x+1)^5}{(2x+1)^5}$

$$= \frac{5(x+1)^4}{(2x+1)^5}\left[(2x+1)^{5/2} - (x+1)(2x+1)^{3/2}\right] = \frac{5(x+1)^4}{(2x+1)^{7/2}}\left[(2x+1)-(x+1)\right]$$

$\Rightarrow \boxed{y' = \frac{5x(x+1)^4}{(2x+1)^{7/2}}}$ (OR use logarithmic differentiation)

4 a) $\int_0^{\pi/2} \frac{\cos 3x}{1+4\sin 3x}\,dx$ let $u = 1+4\sin(3x) \Rightarrow du = 12\cos(3x)$

$= \int_{x=0}^{\pi/2} \frac{1}{12}\frac{du}{u} = \frac{1}{12}\left[\ln|u|\right]_{x=0}^{\pi/2} = \frac{1}{12}\left[\ln|1+4\sin 3x|\right]_0^{\pi/2}$

$= \frac{1}{12}\left[\ln|1+4\sin(\tfrac{3\pi}{2})| - \ln|1+4\sin(0)|\right] = \boxed{\frac{1}{12}\ln(3)}$

b) At $x=-2$ $y = -6 + \int_4^4 \frac{\tan(t-4)}{t}\,dt = -6+0$

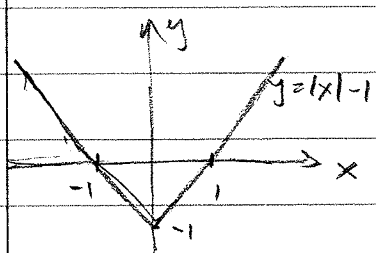
$y'(x) = 3 + \frac{\tan(x^2-4)}{x^2}\cdot\frac{d}{dx}(x^2) = 3 + \frac{2}{x}\tan(x^2-4) \Rightarrow y'(-2) = 3 - \tan(0) = 3$

So the linearization is $y = -6 + 3(x+2) \Rightarrow \boxed{y = 3x}$

c) $\int_{-1}^0 \frac{x^3}{\sqrt{x^4+9}}\,dx = \int_{x=-1}^0 \frac{1}{4\sqrt{u}}\,du$ $u = x^4+9$, $du = 4x^3\,dx$

$= \left[\frac{1}{2}\sqrt{u}\right]_{x=-1}^0 = \frac{1}{2}\left[\sqrt{x^4+9}\right]_{-1}^0 = \frac{1}{2}\left(\sqrt{9}-\sqrt{10}\right) = \boxed{\frac{3-\sqrt{10}}{2}}$

5 a)



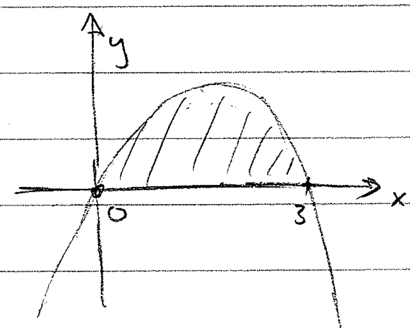
b) $\bar{y} = \frac{1}{3-(-1)}\int_{-1}^3 |x|-1\,dx$

$= \frac{1}{4}\left[\int_{-1}^0 -x-1\,dx + \int_0^3 x-1\,dx\right]$

$= \frac{1}{4}\left(-\left[\frac{x^2}{2}+x\right]_{-1}^0 + \left[\frac{x^2}{2}-x\right]_0^3\right)$

$= \frac{1}{4}\left(\frac{1}{2}-1+\frac{9}{2}-3\right) = \boxed{\frac{1}{4}}$

6.



$\int_a^b 3x-x^2\,dx$ = "area" under curve $y = 3x-x^2$

This is positive for $x \in [0,3]$ & negative otherwise

So to get maximum (positive) value, take

$\boxed{a=0,\ b=3}$

Taking $a<0$ means including negative area; taking $a>0$ means excluding positive area $\Rightarrow a=0$ is optimal. And similarly for b.