APPM 1350 **Midterm 1** **Spring 2007**

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. There are **7 problems***, plus the "Valentine", that can replace any other 15 points problem. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

1. (10 points)
   Is the function even, odd or neither:

   (a) $y = (-x)^{2/3}$ (b) $y = \dfrac{\sin x}{x^2 - 1}$ (c) $y = 1$

2. (15 points)
   (a) Find the domain of $f(x) = \tan\left(x - \dfrac{\pi}{2}\right)$.
   (b) If $x \in [0, \pi/2]$ and $\tan x = 2$, find $\sin x$.

3. (20 points)
   Calculate the following limits. If they do not exist, write DNA.

   (a) $\lim\limits_{x\to 9} \dfrac{\sqrt{x} - 3}{x - 9}$ (c) $\lim\limits_{x\to 1} \cos\left(\dfrac{x^2 - 2x + 1}{x^2 - 1}\right)$

   (b) $\lim\limits_{x\to 2} \sqrt{5 + \sqrt[3]{2x^5}}$ (d) $\lim\limits_{x\to 0} \dfrac{x + 5}{x}$

4. (15 points)
   (a) State the Intermediate Value Theorem (for continuous functions).
   (b) Explain why the equation $\cos x = x$ has a solution in $[0, \pi/2]$
   (c) Does the function $f(x) = \sqrt{x}\cos\left(\dfrac{9}{x}\right)$ have a continuous extension at $x = 0$?

5. (15 points)
   On what intervals is each of the following functions differentiable? In each case, calculate the derivative where is exists.

   (a) $y = (x^2 + 1)(x + 5 + \dfrac{1}{x})$
   (b) $y = |x - 4|$
   (c) $y = 5\cos\left(\dfrac{\pi}{3}\right)$

6. (10 points)

(a) Using the definition, calculate the derivative of $f(x) = \dfrac{1}{x+3}$.

(b) Evaluate $f'(0)$.

(c) Find the equation of the tangent line to the graph of f at the point $\left(0, \dfrac{1}{3}\right)$.

7. (15 points)

A rock climber accidentally kicks a rock loose when she is 576 feet high.

(a) How long does it take the rock to hit the ground?

(b) What is the average velocity at that time?

(c) What is the instantaneous velocity of the rock at the moment it hits the ground?

Joker problem: Imagine your heart as the contour given by the union of the graphs of the two functions $y = |x| + \sqrt{1-x^2}$ (solid line) and $y = |x| - \sqrt{1-x^2}$ (dashed line), for $-1 \leq x \leq 1$. Cupid always shoots arrows horizontally towards your heart. If he hits it centrally, you fall in love. If he hits it tangentially, you get blue. If he misses, he won't next time. Can you tell at what point(s) on the contour has your heart been hit if you start to feel blue?