

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. Solve all 5 problems. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points)

(a) Is there a value of b that would make the function

$$f(x) = \begin{cases} x + b, & x < 0 \\ \cos(x), & x \geq 0 \end{cases}$$

differentiable at $x = 0$?

(b) Calculate: $\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$

(c) Calculate: $\lim_{x \rightarrow -\infty} \frac{x^4 + 3x + 1}{x^2 + 1}$

(d) Use implicit differentiation to find $\frac{dy}{dx}$ in $x^2 = \frac{x - y}{x + y}$

2. (20 points)

A cop is trying to videotape a robber who has stolen a Porche. Unknown to her, there is an automatic camera placed on a pole 132 ft above the ground tracking her movements. The car is moving directly towards the pole at 264 ft/sec. What would the speed of the car with respect to the camera be when the car is 30 ft from the base of the pole?

3. (20 points)

Consider the equation $x = \frac{1}{x^2 + 1}$.

(a) Show that the equation has at least one solution in $(0, 1)$.

(b) Approximate this solution by using Newton's method for $x_0 = 1$ (calculate only x_1).

4. (20 points)

Consider the function $f(x) = \frac{x^2}{x - 1}$, for $x \neq 1$.

(a) Find the x and y coordinates of all local maxima and minima. Identify which are maxima and which are minima. Justify your answers.

(b) Determine any horizontal or vertical asymptotes the graph of f might have.

(c) Graph f , using the information from (a) and (b). In your graph, show and label all maxima, minima and asymptotes.

5. (20 points)

(a) State the Mean Value Theorem.

(b) The US postal service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 in. What dimension will give a box with a square end and the largest possible volume?