

*Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. There are 7 problems, plus the "joker" that can replace any other 15 points problem. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

1. (15 points)

Graph the following functions of  $\theta$ , each on a different coordinate system:

(a)  $y = \sin \theta$       (b)  $y = \sin\left(\frac{\pi}{2} - \theta\right)$       (c)  $y = 1 - \sin(\theta)$

2. (15 points)

Consider the functions  $f(x) = \frac{x}{x-1}$  and  $g(x) = x + 1$ .

- (a) What are the domains of  $f$  and  $g$ ?  
(b) Calculate  $(g \circ f)(x)$ . What is the domain of  $(g \circ f)$ ?  
(c) Calculate  $(f \circ f)(x)$ . What is the domain of  $(f \circ f)$ ?

3. (15 points) Consider the function  $f(t) = 2 \sin\left(\frac{t}{2}\right)$ , defined on the interval  $[0, 4\pi]$ .

- (a) What is the average rate of change of  $f$  over the interval  $[0, \pi/3]$ ?  
(b) What is the instantaneous rate of change of  $f$  at  $t = 0$ ?  
(c) What is the equation of the tangent line at the origin to the graph of  $f$ ?

4. (20 points)

Calculate the following limits. If they do not exist, write DNE.

(a)  $\lim_{x \rightarrow 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$       (c)  $\lim_{x \rightarrow \sqrt{2}} \left(\frac{x^2}{2} - \frac{1}{x}\right)$   
(b)  $\lim_{x \rightarrow 0^-} \frac{4}{x^2/5}$       (d)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$

5. (10 points)

- (a) State carefully the definition of a continuous function.  
(b) For what value of  $a$  is the function

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax & , \quad x \geq 3 \end{cases}$$

continuous at every point?

(c) Is there a value of  $a$  for which  $f$  is differentiable at every point?

6. (15 points)

(a) For  $y = \frac{2x+1}{x^2+1}$ , calculate  $\frac{dy}{dx}$ .

(b) For  $y = x^3 + \frac{1}{x+1}$ , calculate  $\frac{d^2y}{dx^2}$ .

7. (10 points)

(a) Using the definition, calculate the derivative of the function  $y = \sqrt{x+1}$  for  $x > -1$ .

(b) At what point(s) on the curve at point (a) does the tangent line have slope  $m = 2$ ?

Joker problem: (15 points)

We suspect that 3 particles originated as by-products of the same molecular explosion, but we are not certain of what happened at the time of the explosion. The particles move along the same line. Particle  $X$  is oscillating, so that at time  $t$  from its birth it is  $f(t) = \frac{\sin t}{t}$  microns away from a reference point  $O$ . Particle  $Z$  is slowly moving away, so that at time  $t$  it is  $g(t) = t+1$  from  $O$ . All we know about particle  $Y$  is that it is always trapped between  $X$  and  $Z$ . Is our suspicion on their common origin correct? (Hint: You want to check that the past trajectories of all particles approach the same point as  $t$  approaches zero.)