APPM 1350 **Forgiveness exam III** **Spring 2007**

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. There are **5 problems** *of 20 points each, plus a 20-point extra credit problem. Show your work clearly and box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

(1) (a) Estimate the integral $\int_0^3 \sqrt{x+1}dx$ using $n = 6$ rectangles and left-hand end-points (you do not need to simplify your answer).
(b) Estimate the same integral using the trapezoidal sum for $n = 6$ (you do not need to simplify your answer).
(c)How large do you have to make n to be sure that the corresponding trapezoidal estimate is within 10^{-4} of the real value of the integral?

Hint: The trapezoidal sum and error bound for the trapezoidal rule are
$T_n = \frac{h}{2}(y_0 + 2y_1 + ... + 2y_{n-1} + y_n)$
$|E_T| \leq \dfrac{b-a}{12}h^2M$

(2)Find the total area of the two regions enclosed between the graph of the function $y = x^2 - 1$ on the interval $[0, 2]$ and the x-axis.

(3) Calculate the integrals:
(a) $\displaystyle\int \frac{\sqrt[3]{t}+1}{\sqrt{t}}dt$
(b) $\displaystyle\int \frac{(1+\sqrt{x})^3}{\sqrt{x}}dx$
(c) $\displaystyle\int_0^{\sqrt{7}} t(t^2+1)^{1/3}dr$

(4) (a) State the Fundamental Theorem of Calculus (both parts).
(b) Calculate the derivative of the function $h(x) = \displaystyle\int_0^{\sqrt{x}} \cos\theta d\theta$.
(c) Find the critical points of the function h given in part (b).

(5) (a) Find the average value of the function $f(x) = 3x^2 - 3$ on the interval $[0, 1]$.
(b) The Mean Value Theorem states that there is a value $x = c$ such that $f(c)$ equals the average value of f in over some interval. Find this value c for the function and interval in part (a).

Extra credit:

A fence of height H is D feet away from a vertical wall. At what angle θ should a ladder be leaned against the fence in order that the minimum length ladder be required to stretch from the ground to the wall? Use the following guidelines:

(a) Where (how far from the fence) should the leg of the ladder A be placed on the ground so that the ladder makes a given angle θ with the fence? (Hint: your answer will depend on D and θ.)

(b) How far is A from the vertical wall, for the given angle θ?

(c) How long is the ladder for the given θ?

(d) The length L of the ladder depends on θ (via one of its trig functions $\sin\theta$, $\cos\theta$ etc). Differentiate L with respect to θ. (Hint: when differentiating, remember that H and D are constants that do not depend on θ.)

(e) Set the derivative of L equal to zero and solve for θ to find for which angle the minimum value of L is obtained. (Hint: your answer may still depend on H and D. You may express θ in the form of one of its trig functions, e.g. $\cos\theta = 100G + \sqrt[100]{H}$.)

Good luck!