List of exam problems

(1) Soup is being poured at a rate of 3 cubic inches per second into a hemispherical bowl with a radius of 5 inches. How fast is the level of soup rising when there are 2 inches of soup in the bowl?

(2) An athlete is running around a circular track of radius 100m at 5m/sec. A spectator is 300m from the center of the track. How fast is the distance between the two changing when the runner is approaching the spectator and the distance between them is 250m?

(3) A billboard 54 feet wide is perpendicular to a straight road, and is 18 feet east of the nearest point on the road. A car approaches the billboard form the south. From what point does a passenger see the billboard at the widest angle?

(4) A boat is 4 miles from the nearest point on a straight shore line which is 6 miles from a shoreside reastaurant. A woman plans to row to a point on shore, and then walk to the restaurant. If she can walk at 3 mph, at what speed must she be able to row so that the quickest way to get to the restaurant is to row directly?

(5) In constructing the new Trump Colosseum, projected to occupy the entire state of Rhode Island, the builder estimates the initial costs (buying Rhode Island, etc) as 450 times the cost of the first floor. The cost of the second floor is projected to cost twice as much as the first, the third floor three times as much as the first floor, etc. What number of floors in the building will give the cheapest average cost per floor?

(6) A fence of height H is D feet away from a vertical wall. At what angle θ should a ladder be leaned against the fence in order that the minimum length ladder be required to stretch from the ground to the wall?

Optional problems

I. A truck driving over a flat interstate at a constant speed of 50mph gets 4 miles to the gallon. Fuel costs \$ 0.89 per gallon :) For each mile per hour increase in speed, the truck loses a tenth of a mile per gallon in its mileage. Drivers get \$ 27.50 per hour in wages, and fixed costs for running the truck amount to \$ 11.33 per hour. What constant speed (between 50mph and the speed limit of 65mph) should a dispatcher require on a straight run through 260 miles of Kansas interstate to minimize the total cost of operating the

truck?

II. A wheel of radius 10 cm is revolving 4 times per second. Attached to the rim is one end of a rod of length 30 cm. The other end of the rod moves a piston in a straight line extending through the center of the wheel. How fast is the piston moving when the angle at A is a right angle? (See attached picture.)

III. Financial reversals force Elms Lea Inn to convert a hotel into an apartment house with 65 rental units. At $ 300, all the units can be rented. For each $ 10 that the rent is raised, one of the units becomes vacant. Each occupied unit requires $ 30 in service each month. How much rent should be charged to maximize cash flow?

IV. A rectangle has length L and width W. What is the area of the largest rectangle that can be circumscribed around it? (See picture.)