APPM 1350: Exam 3 Suggested Review Problems

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APPM 1350: Exam 3 Review - Important Concepts

1. If $u(x)$ is any differentiable function and $n \in \mathbb{R}$,

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + C$$

2. Definition of the definite integral in terms of Riemann sums

$$\int_a^b f(t)\,dt = \lim_{\|P\| \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k$$

$\Delta x_k = x_k - x_{k-1}$

$c_k \in [x_{k-1}, x_k]$

$a = x_0 < x_1 < x_2 < \dots < x_n = b$

3. Approximation of the definite integral using the rectangle rule

$$\int_a^b f(t)\,dt \approx \sum_{k=1}^{n} f(c_k)\,\Delta x_k$$

4. If f is integrable on $[a,b]$, the average value of $f(x)$ on $[a,b]$ is

$$f_{avg} = \frac{1}{b-a} \int_a^b f(t)\,dt$$

5. The mean value theorem for definite integrals. If f is continuous on $[a,b]$, then at some point $c \in [a,b]$,

$$f(c) = \frac{1}{b-a} \int_a^b f(t)\,dt$$

MEMORIZE!

6. The fundamental theorem of calculus

Part 1: If f is continuous on $[a,b]$, then $F(x) = \int_a^x f(t)$ has a derivative at every point of $[a,b]$ and

$$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x) \qquad a \le x \le b$$

Part 2: If f is continuous at every point of $[a,b]$ and F is any antiderivative of f on $[a,b]$, then

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

7. Substitution in definite integrals. If g is a function of x, then

$$\int_a^b f\circ g\,dx = \int_{g(a)}^{g(b)} f(u)\,du$$

8. The trapezoidal rule.

$$\int_a^b f(t)\,dt \approx \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$$

where $a = x_0 < x_1 < \cdots < x_n = b$ where $\Delta x_k = x_k - x_{k-1} = \frac{b-a}{n} = h$ and $y_k = f(x_k)$.

If f'' is continuous and $M \ge |f''(x)|$ for $x \in [a,b]$, then the error using the trapezoidal rule is bounded by

$$|E_T| \le \frac{b-a}{12} h^2 M = \frac{(b-a)^3 M}{12 n^2}$$

To insure an error no larger than ϵ, we require

$$\frac{(b-a)^3 M}{12 n^2} \le \epsilon \Rightarrow n \ge \sqrt{\frac{(b-a)^3 M}{12\epsilon}}$$ (rounded up to the nearest integer)

9. Derivative rule for inverses. If f is differentiable at every point of an interval I and $df/dx \neq 0$ on I, then f^{-1} is differentiable at every point of the interval $f(I)$, provided f^{-1} exists. Further

$$\left(\frac{df^{-1}}{dx}\right)_{x=f(a)} = \frac{1}{\left(\frac{df}{dx}\right)_{x=a}}$$

Put differently; if $y = f(x)$ then $x = f^{-1}(y)$ and

$$\frac{dx}{dx} = \frac{1}{\left(\frac{dy}{dx}\right)}$$

10. Natural logarithms

$$\ln x = \int_1^x \frac{dt}{t} \qquad (x > 0)$$

$$\frac{d}{dx} \ln u = \frac{1}{u}\frac{du}{dx} \qquad (u > 0)$$

$$\int \frac{du}{u} = \ln|u| + c \qquad (u \neq 0)$$

11. Exponentials

$$e = \ln^{-1} 1$$

$$e^{\ln x} = x = \ln e^x$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\int e^u\, du = e^u + c$$

12. Properties of logs

$$\ln ax = \ln a + \ln x \qquad (a, x > 0)$$

$$\ln \frac{a}{x} = \ln a - \ln x \qquad (a, x > 0)$$

$$\ln x^n = n \ln x \qquad (x > 0)$$

13. Laws of exponents for e^x

$$e^{x_1} e^{x_2} = e^{x_1 + x_2}$$

$$e^{-x} = \frac{1}{e^x}$$

$$\frac{e^{x_1}}{e^{x_2}} = e^{x_1 - x_2}$$

$$(e^{x_1})^{x_2} = e^{x_1 x_2} = (e^{x_2})^{x_1}$$

14. Rules for definite integrals

$$\int_a^a f(t)\,dt = 0$$

$$\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$$

$$\int_a^b k f(t)\,dt = k \int_a^b f(t)\,dt \qquad (k = \text{constant})$$

$$\int_a^b [f(t) \pm g(t)]\,dt = \int_a^b f(t)\,dt \pm \int_a^b g(t)\,dt$$

$$\int_a^c f(t)\,dt + \int_c^b f(t)\,dt = \int_a^b f(t)\,dt$$

$$(b-a) f_{min} \le \int_a^b f(t)\,dt \le (b-a) f_{max} \quad \text{where} \quad f_{min} = \min_{x \in [a,b]} f(x), \quad f_{max} = \max_{x \in [a,b]} f(x)$$

$$\text{If } f(x) \ge g(x) \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$$