APPM 1350 **Midterm III** **Spring 2008**

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name (Radulescu or Chang) and (3) recitation number on the front of your bluebook. There are **6 problems**, *plus an extra credit problem. Show your work clearly and box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.*

1. (20 points)

   (a) Estimate $\displaystyle\int_0^1 \sin(\pi t)\,dt$ using the trapezoidal sum for $n = 4$.

   (b) How large would you have to make n to be sure that the corresponding trapezoidal estimate is within 0.01 of the real value of the integral?

   Hint: The trapezoidal sum and error bound for the trapezoidal rule are:

   $T_n = \frac{h}{2}(y_0 + 2y_1 + ... + 2y_{n-1} + y_n), \quad |E_T| \leq \dfrac{b-a}{12}h^2M$

2. (15 points) Find the total area of the two regions enclosed between the graph of the function $y = 1 - \dfrac{x^2}{4}$ on the interval $[-2, 3]$ and the x-axis.

3. (20 points) Calculate the integrals:

   (a) $\displaystyle\int \frac{(1+\sqrt{t})^{1/2}}{\sqrt{t}}\,dt$

   (b) $\displaystyle\int \frac{\pi}{2}(\cos x)\sin(\pi + \pi\sin x)\,dx$

   (c) $\displaystyle\int_{-1}^{2} \frac{t\,dt}{\sqrt{2t^2+8}}$

4. (15 points)

   (a) State the Fundamental Theorem of Calculus (both parts).

   (b) Suppose $\displaystyle\int_1^x f(t)\,dt = x^2 - 2x + 1$. Find $f(x)$.

5. (15 points)

   (a) Find the average value of the function $f(x) = (x-1)^2$ on the interval $[0, 3]$.

   (b) The Mean Value Theorem states that there is a value $x = c$ such that $f(c)$ equals the average value of f. Find this value c.

6. (15 points) Using logarithmic differentiation, calculate the derivative of:

$$y = \sqrt[3]{\frac{x(x-2)}{x^2+1}}.$$

Extra-credit: (20 points)

A boat is 3 miles from the nearest point on a straight shore line which is 5 miles from a shoreside restaurant. A woman plans to row to a point on shore, and then walk to the restaurant. If she can walk at 4 mph, at what speed must she be able to row so that the quickest way to get to the restaurant is to row directly? You may use the following guidelines:

(a) Call v the unknown rowing speed.

(b) Suppose the woman decides to row to a point on the shore x miles away from the restaurant. What is the distance d_1 she will be rowing? (Hint: Use the Pythagorean Theorem.) What is the distance d_2 she will be walking?

(c) What is the time T_1 spent rowing with the speed v? What is the time T_2 spent walking at 4 mph? What is the total time $T = T_1 + T_2$? (Hint: your answer should depend on v and x.)

(d) The total time T depends on the choice of x. Find when T is minimal by differentiating with respect to x and calculating the critical point. (Hint: When differentiating, think of v as a constant, that does not depend on x. Your answer will still depend on the speed v.)

(e) Find the value of v by setting the critical point found at (d) to be equal to 5 mi.

Good luck!