APPM 1350 **Forgiveness midterm II** **Spring 2008**

INSTRUCTIONS: Books, notes, flying monkeys and electronic devices are not permitted. Write your (1) name, (2) student number, (3) instructor's name (Radulescu or Chang), and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. Work all 6 problems. Start each problem on a new page. Show your work. **BOX** in your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points)

(a) Is there a value of b that would make the function

$$f(x) = \begin{cases} x + b, & x < 0 \\ \cos(x), & x \geq 0 \end{cases}$$

differentiable at $x = 0$?

(b) Calculate: $\lim_{x \to 0} x \cot x$

(c) Calculate: $\lim_{x \to \infty} \dfrac{x^3 + 5x + \sin x}{x^2 + 1}$

2. (10 points) Use implicit differentiation to find $\dfrac{dr}{d\theta}$ for the curve defined by $\cos r + \cos \theta = r\theta$.

3. (20 points) A spherical iron ball 8 in. in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 10 in^3/min, how fast is the thickness of the ice decreasing when it is 2 in. thick? How fast is the outer surface area of the ice decreasing?

4. (20 points) Consider the equation $x = \dfrac{1}{x^2 + 1}$.

(a) Show that the equation has at least one solution in $(0, 1)$.

(b) Approximate this solution by using Newton's method for $x_0 = 1$ (calculate only x_1).

(c) Sketch a graph showing how you obtained x_1 from the initial guess x_0.

5. (20 points) Consider the function $f(x) = \dfrac{x^2 + 4x + 4}{x^2 - 4}$, for $x \neq \pm 2$.

(a) Find the x and y coordinates of all local maxima and minima. Identify which are maxima and which are minima. Justify your answers.

(b) Determine any horizontal or vertical asymptotes the graph of f might have.

(c) Graph f, using the information from (a) and (b). In your graph, show and label all maxima, minima and asymptotes.

6. (20 points)

(a) State the chain rule.

(b) I wish to build pens for five dogs out of 240 feet of fence. I'll build the pens adjacent to each other against a house, so one side doesn't need fencing. What dimensions will give the dogs the most room to roam?