

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. You need to solve all 5 problems in order to receive full credit. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points) Establish whether the following statements are True or False. If your answer is "False," justify your reasoning by means of a counter-example.

(a) The equation

$$x = \frac{1}{x^2 + 1}$$

has at least one solution in $(0, 1)$.

(b) If f and g are differentiable and $f' = g'$ on (a, b) , then $f = g$ on (a, b) .

(c) f is increasing at all points where $f' > 0$.

2. (15 points)

(a) Calculate the following limit. Please, carefully justify your answer. If the limit doesn't exist, explain why.

$$\lim_{x \rightarrow \infty} \frac{x^2 + x + \sin x}{5x^2 + 1}$$

(b) Use implicit differentiation to find the tangent line at the point $(1, 0)$ to the curve defined by the equation below:

$$x^2 = \frac{x - y}{x + y}$$

3. (15 points)

(a) Calculate the linearization of the function $f(x) = \cos(\sqrt[3]{x+1}\frac{\pi}{2})$ at the point $(0, 0)$.

(b) Use this linearization to estimate the value of $\cos(\sqrt[3]{1.1}\frac{\pi}{2})$

4. (25 points) Consider the function:

$$f(x) = \frac{x^2 + 3}{x - 1}, \quad x \neq 1$$

(a) Find the x and y coordinates of all local maxima and minima. Identify which are maxima and which are minima. Justify your answers.

(b) Determine any horizontal, vertical and oblique asymptotes the graph of f might have.

(c) Using the results from (a) and (b), sketch the graph of f . In your graphs, show and label the maxima, minima and asymptotes.

5. (20 points) A man wants to enclose three identical rectangular pens for his three chickens using 10 yards of fence in total. He is planning to have the pens adjacent to each other, and all adjacent to his house wall, as shown in the figure below. What dimensions does each pen need to have in order to maximize the area that each chicken will have? (No fence is needed along the house wall.)

