

Books, notes and electronic devices are not permitted. Write your (1) name, (2) instructor's name and (3) recitation number on the front of your bluebook. There are **6 problems**, plus the extra credit problem that can replace any other 15 point problem. Show your work clearly and box your answer. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (15 points)

Graph the following functions of  $\theta$ , each on a different coordinate system:

(a)  $y = \sin \theta$       (b)  $y = \sin\left(\frac{\pi}{2} - \theta\right)$       (c)  $y = 1 - \sin(\theta)$

2. (15 points)

Consider the functions  $f(x) = \frac{x}{x-1}$  and  $g(x) = x + 1$ .

- (a) What are the domains of  $f$  and  $g$ ?  
(b) Calculate  $(g \circ f)(x)$ . What is the domain of  $(g \circ f)$ ?  
(c) Calculate  $(f \circ f)(x)$ . What is the domain of  $(f \circ f)$ ?

3. (15 points)

Consider the function  $y = \sqrt{1-x}$ , defined on the interval  $[-1, 1]$ .

- (a) What is the average rate of change of  $f$  over the interval  $[0, 1]$ ?  
(b) Using the definition of the derivative, calculate the instantaneous rate of change of  $f$  at  $x = 0$ ?  
(c) What is the equation of the tangent line at the origin to the graph of  $f$ ?  
(d) At what point(s) does the tangent line have slope  $m = 1$ ?

4. (20 points)

Calculate the following limits. If they do not exist, write DNE.

(a)  $\lim_{x \rightarrow 1^+} \frac{\sqrt{2x}(x-1)}{|x-1|}$       (c)  $\lim_{x \rightarrow \sqrt{2}} \left( \frac{x^2}{2} - \frac{1}{x} \right)$   
(b)  $\lim_{x \rightarrow 0^-} \frac{4}{x^2/5}$       (d)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$

5. (20 points)

(a) For  $y = \frac{2x+1}{x^2+1}$ , calculate  $\frac{dy}{dx}$ .

(b) For  $y = x^3 + \frac{1}{x+1}$ , calculate  $\frac{d^2y}{dx^2}$ .

**6.** (15 points)

Below is the graph of a function over a closed interval  $D$ . At what domain point does the function appear to be:

- (a) differentiable?
- (b) continuous but not differentiable?
- (c) neither continuous nor differentiable?

**Extra credit.** (15 points)

- (a) State carefully the definition of a continuous function.
- (b) For what value of  $a$  is the function

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax & , \quad x \geq 3 \end{cases}$$

continuous at every point?

- (c) Is there a value of  $a$  for which  $f$  is differentiable at every point?