

## EXAM #3

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your recitation time and recitation instructor, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A one-page crib sheet is allowed.

1. (15 points) Consider the function  $f(x) = x^3 + x - 3$  on the closed interval  $[-1, 2]$ .
  - (a) Explain why there is one and only one real solution of  $f(x) = 0$ .
  - (b) Sketch the curve. Use Newton's method to estimate the solution of  $f(x) = 0$  to within accuracy of 0.01.
2. (15 points) For each of the functions listed below, given a positive value for  $\epsilon$ , find the value of  $\delta$  (in terms of  $\epsilon$ ) such that  $0 \leq |x - x_0| \leq \delta \Rightarrow |f(x) - L| \leq \epsilon$ . Assume that  $\epsilon$  is a small number, say  $\epsilon \ll 1$ .
  - (a)  $f(x) = \frac{x^2 - 4}{x - 2}$  and  $x_0 = 2$
  - (b)  $f(x) = \sqrt{x + 1}$  and  $x_0 = 3$
  - (c)  $f(x) = x^2 - 5$  and  $x_0 = 4$
3. (15 points) Consider the curve  $y = \sin(\pi x)$ .
  - (a) Calculate the slope of the secant line connecting the points on the curve corresponding to  $x = 0$  and  $x = 1/2$ .
  - (b) Write the equation of the secant line.
  - (c) Write the equation of the line perpendicular to the secant line, and passing through the origin.
4. (15 points) Calculate the following limits. If the limit does not exist, clearly state that fact.
  - (a)  $\sqrt{x} \sin\left(\frac{1}{x}\right)$  as  $x \rightarrow 0$
  - (b)  $\frac{\sin\left(\frac{1}{x}\right)}{x}$  as  $x \rightarrow 0$
  - (c)  $\lim_{x \rightarrow 0} \sqrt{|x|}$
  - (d)  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x^2 - 4}$
5. (15 points) The last problem I have involves limits/continuity for a function that can be drawn by hand.