APPM 1350 **FINAL EXAM** **FALL 1996**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A one-page crib sheet is allowed. Please start each new problem on a new page of the bluebook.

1. (20 points) Evaluate each of the following limits, if the limit exists. If the limit does not exist, state this. Explain your reasoning in each case!

a) $\displaystyle\lim_{t\to\infty} \frac{2 - t + \sin t}{2t + \cos t}$

b) $\displaystyle\lim_{x\to 1} \frac{x^2 + x - 2}{x - 1}$

c) $\displaystyle\lim_{h\to 0} \frac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h}$

2. (20 points) Determine whether each of the following statements is true or false. For each statement, if it is true, give a reason why it is true. If it is false, either give a reason why it is false, or give a counterexample showing that it is false.

a) $\displaystyle\frac{d}{dx}\int_0^x f\Big(g(u)h(u)\Big)\,du = f\Big(g(x)h(x)\Big)\; g'(x)h'(x)$

b) If an odd function $f(x)$ has a local minimum value at $x = c$, then f also has a local minimum at $x = -c$.

c) Suppose that $y = f(x)$ is continuous on a closed interval $[a, b]$ and differentiable on the interval (a, b). Then there is at least one point c in (a, b) at which $\dfrac{f(b) - f(a)}{b - a} = f'(c)$.

d) $\displaystyle\int_a^b |f(x)|\,dx = \left|\int_a^b f(x)\,dx\right|$

e) Newton's method will always find the roots of $f(x) = 0$, as long as $f'(x)$ exists everywhere.

3. (20 points) Consider the function $y(x) = \dfrac{1}{1+x^2}$. Use calculus to do the following:

a) Find the x and y coordinates of any local maxima and local minima.

b) Find the inflection points and intervals of concavity.

c) Neatly graph the function and clearly indicate any asymptotes.

4. (20 points) Find $\dfrac{dy}{dx}$ for the functions in parts (a), (b) and (c). Then do part (d).

a) $y = \sin\left(\pi - x^{-1}\right)$

b) $\displaystyle y = \int_0^x \sec(\sin u)\,du$

c) y is defined by $x + y = \sin y$.

d) Calculate $\dfrac{d^2y}{dx^2}$ from your answer to part (c). Your final answer should be in terms of x and y only.

5. (25 points) A round hole of radius $\sqrt{3}$ feet is bored through the center of a solid sphere of radius 2 feet. Find the volume of material removed from the sphere.

6. (25 points) A 4 meter length of wire is available for making a circle and a square.

a) How should the wire be distributed between the two shapes to *maximize* the total enclosed area? How do you know that you have found a gobal maximum?

b) How should the wire be distributed between the two shapes to *minimize* the total enclosed area? How do you know that you have found a gobal minimum?

7. (20 points) A large irregularly shaped sheet of material is shown in Figure 1 below. Use Simpson's Rule to approximate the area. The dimensions are in feet and the total horizontal width across the entire piece of material is 8 feet. The vertical measurements were recorded at uniform intervals of 1 foot.

8. (25 points) A funnel has the shape of an inverted cone with a top radius of 5 inches and a height of 15 inches, as shown in Figure 2 below. Water flows out of the bottom of the funnel at a constant rate of 27 inches^3/minute. At what rate is the depth of the water changing at the instant that the depth is 9 inches? Be sure to include units in your work!

9. (25 points) Consider a flat, semi-circular plate of radius R. Suppose the density of the plate, δ, is constant. Find the center of mass of the plate.

10. (Extra Credit, 20 points) A stone is thrown into a still pond and a ripple is set off. Let $h(t)$ be the height of the ripple, as measured from the flat surface of the rest of the pond. At any given instant, the rate of change of the ripple height, $\frac{dh}{dt}$, is proportional to the square root of the current height of the ripple. Find the ripple height as a function of time, given that at $t = 0$, $h(0) = 1$, and $\frac{dh}{dt}(0) = 2$.

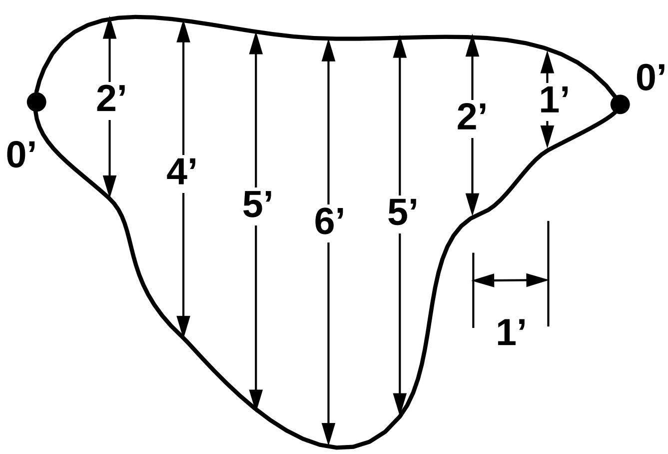

Figure 1

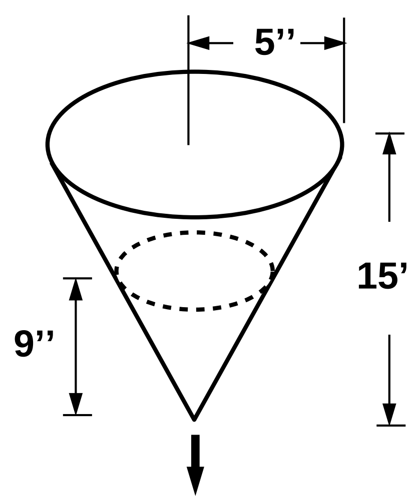

Figure 2