APPM 1350 FINAL EXAM Spring 1997

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (25 points) Evaluate the following limits, if they limit exists. If the limit does not exist, state this. Explain your reasoning in each case.

   (a) $\lim_{x\to 0} \dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h}$

   (b) $\lim_{x\to 0} x \tan x$

   (c) $\lim_{x\to\infty} (x + 1 - \sqrt{x^2+1})$

   (d) $\lim_{x\to 4} \left(\dfrac{x^2-6x+8}{x^2-7x+12} + \dfrac{1}{x} \right)$

   (e) $\lim_{x\to 0} \dfrac{1}{x} \displaystyle\int_0^x \cos^2 t \, dt$

2. (20 points) Find $\dfrac{dy}{dx}$ for each of the following functions using the appropriate differentiation rules. *Do not* simplify your result.

   (a) $y = \dfrac{1}{3x^2} - \dfrac{5x}{2}$

   (b) $y = (x^2+1)(x^2+x+1)$

   (c) $y = \tan(\sin x)$

   (d) $y = \displaystyle\int_{3\sqrt{x}}^{5} \frac{t^2-2}{t-1}\, dt$

3. (25 points) Evaluate the following integrals.

   (a) $\displaystyle\int \cos^2 3x \, dx$

   (b) $\displaystyle\int \frac{\tan x}{\cos^2 x}\, dx$

   (c) $\displaystyle\int_0^{2\pi} |\sin x|\, dx$

   (d) $\displaystyle\int_0^{\frac{\pi}{2}} 3\sin x \, dx$

   (e) $\displaystyle\int_0^1 \sqrt{7}\, dx$

   (f) $\displaystyle\int_0^2 \frac{4y}{\sqrt{2y^2+1}}\, dy$

   (g) $\displaystyle\int_0^{\frac{\pi}{4}} \sin^4 r \cos r \, dr$

4. (25 points) Consider the function $f(x) = \sqrt{x^2 + 4}$.

   (a) Find the domain and range of this function.

   (b) Find all the asymptotes of this function.

   (c) Find the local extreme values of the function.

   (d) What are the concavity intervals?

   (e) Graph the function and clearly indicate any results you obtain in parts (a) through (d).

5. (25 points) Consider the curve defined by $(x^2 + y^2)^2 - 4xy = 0$. Call this curve 'A'.

   (a) Find all the points where the curve 'A' intersects the line $y = x$.

   (b) Construct the tangent line to curve 'A' at the point $(1, 1)$.

   (c) Construct the normal line to the curve 'A' at the point $(1, 1)$.

   (d) Draw the tangent line and the normal line at the point $(1, 1)$.

   (e) Find the linearization of curve 'A' at the point $(1, 1)$.

6. (10 points) Consider a sphere for which the radius is decreasing at a constant rate of 2 feet per second. At the instant the radius of the sphere is 4 feet, at what rate is the volume changing?

7. (10 points) A particle has an acceleration of $a = \frac{3}{8}t$ feet per second squared. At the time $t = 4$ seconds, the particle's position is $s = 4$ feet, and the particle's speed is $v = 3$ feet per second.

   (a) Calculate the particle's speed as a function of time $v(t)$.

   (b) Calculate the particle's position as a function of time $s(t)$.

8. (20 points) A drilling rig 12 miles offshore is to be connected by a pipe to a refinery onshore, 20 miles down the coast from the rig. If underwater pipe costs \$50,000 per mile and land-based pipe costs \$30,000 per mile, what values of x and y give the least expensive connection?

9. (25 points) Consider the spherical loaf of bread shown in the figure. The radius of the loaf is 5 inches. Suppose a slice, perpendicular to the x-axis, is cut from the loaf, as shown in the figure. The left-hand cut is located at a position a on the x-axis and the right-hand cut is located 1 inch to the right, at the position $a + 1$ on the x-axis. What is the surface area of the crust on the slice of bread?

10. (20 points) Consider the integral $\int_0^6 x^2\,dx$.

    (a) Numerically evaluate the integral using Simpson's rule with $h = 1$.

    (b) Compare your result from part (a) to the exact value of the integral and discuss the results.

11. (20 points extra credit) Consider a sphere of radius R, where R is greater than 6 inches. A hole is drilled through the center of the sphere, as shown in the figure below. The height of the remaining section of the sphere is 6 inches. Find the volume of material remaining.