

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. Crib sheets and calculators are not allowed. Please start each new problem on a new page of the bluebook.

1. (18 points) Evaluate each of the following limits, if the limit exists. If the limit does not exist, state this. Explain your reasoning in each case!

(a) $\lim_{x \rightarrow 2} \sqrt{x-2}$

(b) $\lim_{x \rightarrow 2^-} \frac{x}{x^2 - 4}$

(c) $\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

2. (15 points) Determine whether each of the following statements is true or false. For each statement, if it is true, give a reason why it is true. If it is false, either give a reason why it is false, or give a counterexample showing that it is false.

(a) If f is an odd function and $\lim_{x \rightarrow 0^+} f(x) = 3$ then $\lim_{x \rightarrow 0^-} f(x) = 3$.

(b) If $\lim_{x \rightarrow 2} \frac{f(x) - 3}{x - 1} = 5$ then $\lim_{x \rightarrow 2} f(x) = 5$.

(c) Given that $\lim_{x \rightarrow 0} (1+x)^x$ is a positive finite value (you don't need to know what the value is), we can then conclude that $\lim_{x \rightarrow 0} (1+x)^{2x}$ is also a positive finite value.

3. (15 points) Consider the function $y = \sin(\pi x)$.

(a) Calculate the slope of the secant line connecting the points on the graph corresponding to $x = 0$ and $x = 1/2$.

(b) Write the equation of the secant line.

(c) Write the equation of the line perpendicular to the secant line, and passing through the origin.

4. (26 points) Let $f(x) = \sqrt{x}$.

(a) What is the domain and range of $f(x)$?

(b) Using the definition of the derivative, find $f'(x)$. Show your work.

(c) Find the equation of the straight line tangent to $y = f(x)$ at the point $x = 4$.

(d) Sketch the graph of $f(x) = \sqrt{x}$ and its tangent line calculated in part (c). Label your graph.

5. (14 points) Consider the piecewise function given by

$$f(x) = \begin{cases} \frac{1}{(x+1)^2} & \text{if } x \leq 1 \\ ax + 4 & \text{if } 1 < x \leq 4 \\ x^2 - 10 & \text{if } 4 < x \end{cases}$$

- (a) Find the value of the parameter a that makes $f(x)$ continuous at $x = 1$.
- (b) Does this value of a also make the function continuous at $x = 4$? Explain.
6. (12 points) A student lives on College Ave. and bikes East one quarter of a mile to the hill in 2 minutes, stops for 10 minutes in a cafe for a quick cup of coffee. Then the student goes West past their house and continues one quarter mile West to the College Grocery in 5 minutes, spends 5 minutes shopping and finally arrives home 2 minutes after leaving the store.
- (a) Draw the graph of $s(t)$, the position of the student from home, as a function of time. On your graph, be sure to use the convention that East of the student's home is positive, and West of the student's home is negative.
- (b) Calculate the average velocity from the cafe to the grocery store based on the time from when the student *leaves* the cafe until the time the student *arrives* at the grocery store.