APPM 1350 **FINAL EXAM** **FALL 1997**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name (Dougherty, Easton, Lomeli, Norris) (4) a grading table. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks and class notes are NOT permitted. One handwritten crib sheet and a calculator are allowed. Please start each new problem on a new page of the bluebook.

1. (20 points) Evaluate $\dfrac{dy}{dx}$ for the functions given below.

   (a) $y = \tan^{-1}(\ln x)$

   (b) $y = x^{\sqrt{x}}$

   (c) $e^x \ln(y) = x^5 y^3$

2. (20 points) Evaluate the following integrals. Show all work. Correct evaluation of an integral on your calculator is worth 2 points.

   (a) $\displaystyle\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$

   (b) $\displaystyle\int \frac{3x}{9+x^4}\,dx$

   (c) $\displaystyle\int_1^e \frac{(\log_3 x)^2 \log_7 x}{x}\,dx$

3. (20 points) Listed below are some functions and some figures. Assume that a and b are positive constants. For each function, match the number of the figure that could be a plot of the function. Your answers should have the following form: e-1, f-9, etc.

   (a) $y = \displaystyle\int_1^x \frac{dt}{t}$

   (b) $y = ax + b\sin x$

   (c) $y = ae^{bx}$

   (d) $y = ae^{-x}\sin x$

   (e) $\sec^{-1}(x-a)$

4. (20 points) Evaluate each of the following limits, if the limit exists. If the limit does not exist, state this. Explain your reasoning in each case! The correct answer with no supporting work is worth zero points, however, a correct evaluation with demonstrated work from a calculator gets 2 points.

   (a) $\lim_{x\to 2} \dfrac{1}{x-2}$

   (b) $\lim_{x\to 0} (1+3x)^{1/x}$

   (c) $\lim_{x\to 2} \left(\dfrac{1}{x-2} - \dfrac{2}{(x-2)^2} \right)$

5. (20 points) Determine whether each of the following statements is true or false. For each statement, if it is true, give a reason why it is true. If it is false, either give a reason why it is false, or give a counterexample showing that it is false.

   (a) If f is an even function and $\lim_{x\to -2^-} f(x) = 3$, then $\lim_{x\to 2^+} f(x) = 3$.

   (b) $\tan(\sin^{-1} x) = \dfrac{1}{\sqrt{1-x^2}}$

   (c) $\lim_{x\to 0} \dfrac{1}{x} \displaystyle\int_0^x 3e^{-(2t^2)}\, dt$ does not exist.

6. (20 points) A continuous function $y = f(x)$ generates the following table of data.

x	0.0	0.1	0.3	0.5	0.7	0.9	1.1
$f(x)$	0	2	3	1	5	4	6

   (a) Approximate $\displaystyle\int_{0.1}^{0.9} f(x)\, dx$ using the trapezoidal rule.

   (b) Estimate the error, assuming that $|f''(x)| < 5$ for $0 \le x \le 1$.

7. (20 points) The following information is known about a function $y = f(x)$.

   - f is continuous on the interval $[0, 8]$
   - $f(0) = f(8) = 0$
   - $f(3) = f(7)$
   - $f'(x) > 0$ on $(0, 3)$ and $(5, 7)$
   - $f'(x) < 0$ on $(3, 5)$ and $(7, 8)$
   - $f''(x) > 0$ on $(0, 2)$ and $(4, 6)$
   - $f''(x) < 0$ on $(2, 4)$ and $(6, 8)$

   (a) Draw the graph of $y = f(x)$ on the interval $[0, 8]$.

   (b) Label the inflection points.

8. (20 points) A television camera at the ground level is filming the lift-off of a space shuttle that is rising vertically according to the position equation $s = 50t^2$, where s is measured in feet and t is measured in seconds. The camera is 200 feet from the launchpad. Find the rate of change in the angle of elevation of the camera at 10 seconds after lift-off.

9. (20 points) The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

10. (20 points) A line segment has end points on the positive x and y axis, and passes through the fixed point $(8, 1)$ as pictured. What is the least length of such a segment?