APPM 1350 **FINAL EXAM** **FALL 1998**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name (Li, Norris) (4) a grading table. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks and class notes are NOT permitted. One handwritten crib sheet and a calculator are allowed. Please start each new problem on a new page of the bluebook.

1. (20 points) Evaluate each of the following limits, if the limit exists. If the limit does not exist, state this. Explain your reasoning in each case! The correct answer with no supporting work is worth zero points,

   (a) $\lim_{x\to 0} \dfrac{x + x\cos x}{\sin x \ \cos x}$

   (b) $\lim_{x\to 0} \sin(x)\cos\left(\dfrac{1}{x}\right)$

   (c) $\lim_{x\to\infty} x^{1/x}$

   (d) $\lim_{h\to 0} \dfrac{\displaystyle\int_0^{\pi+h} e^{\sin t}\,dt - \int_0^{\pi} e^{\sin t}\,dt}{h}$

2. (20 points) Evaluate $\dfrac{dy}{dx}$ for the functions given below.

   (a) $y = (\ln x)^{\ln x}$
   (b) $y = \sin^{-1}(\cos x)$
   (c) $x + \tan(xy) = 0$
   (d) $y = \displaystyle\int_0^{x^2} (t^2 + 1)\,dt$

3. (20 points) Evaluate the following integrals. Show all work.

   (a) $\displaystyle\int \cos(x)\ \cos^2(\sin x)\ dx$

   (b) $\displaystyle\int_0^{\pi} e^{\sin t}\cos t\ dt$

   (c) $\displaystyle\int_{\pi/4}^{\pi/2} \sin(x)(\cos^3(x) + 1)\ dx$

   (d) $\displaystyle\int \frac{dx}{x^2 + 2x + 2}$

4. (20 points) Consider the function $y = 3x^5 - 5x^3 + 100$ on the closed interval $[-2, 2]$.

   (a) Find all critical points.
   (b) Determine all the local maximum and minimum values of the function.
   (c) Determine the absolute maximum and minimum values.
   (d) Are there any inflection points?

5. (20 points)

   (a) Use Newton's method to estimate the zero of the function $f(x) = \sin x - 0.9$. Start with $x_0 = 0$ and find x_1.

   (b) Estimate the value of the integral $\displaystyle\int_0^{\pi} x\sin^2(x)\,ds$ using the trapezoidal rule with four subintervals.

6. (20 points) Determine whether each of the following statements is true or false. For each statement, if it is true, give a reason why it is true. If it is false, either give a reason why it is false, or give a counterexample showing that it is false.

   (a) Since $\lim_{x\to c} \dfrac{x^2-9}{x-3}$ exists for all real numbers c, the function $f(x) = \dfrac{x^2-9}{x-3}$ is continuous for all real numbers.

   (b) If f is an even function and $\lim_{x\to 0^+} f(x) = 3$ then $\lim_{x\to 0} f(x) = 3$.

   (c) x^2 grows at the same rate as $\sqrt{x^4+5x}$

   (d) Since x^2 is continuous everywhere, then $\dfrac{1}{x^2}$ is continuous everywhere.

7. (20 points) The following information is known about a function $y = f(x)$. $f(1) = 0$, $f'(1) = 5$, $f(4) = 2$, and $f'(4) = 3$. Determine the following:

   (a) $f^{-1}(0)$

   (b) $f^{-1}(2)$

   (c) $\dfrac{df^{-1}}{dx}$ at $x = 0$

   (d) $\dfrac{df^{-1}}{dx}$ at $x = 2$

8. (20 points) Consider the function $y(x) = x^2 + 2$.

   (a) Use the definition to find the derivative of $y(x)$.

   (b) Find the equation of the tangent line that passes through the point $(1, 3)$.

   (c) Find the equation of the line which passes through $(1, 3)$ and is perpendicular to the tangent line.

9. (20 points) When Galileo dropped a cannonball from the tower of Pisa, 160 ft above the ground, the ball's height aboveground t seconds into the fall was $s(t) = 160 - 16t^2$.

   (a) What were the ball's velocity, speed and acceleration at time t?

   (b) About how long did it take the ball to hit the ground?

   (c) What was the ball's velocity at moment of impact?

10. (20 points) Your iron works has contracted to design and build a 500-ft^3, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding $\frac{1}{2}$-in.-thick stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weigh as little as possible. What dimensions do you tell the shop to use? Note: when calculating the surface area, or volume, of the container, you may neglect the thickness of the material.