APPM 1350 **EXAM 3** **FALL 1999**

On front of your bluebook, write your NAME, student ID, and TIME of your lecture.

SHOW YOUR WORK: Correct answers with no work shown may receive no credit! Incorrect answers with correct work shown will receive partial credit! **All problems are worth 20 points.**

1. Consider $f(x) = 2x - x^2$ on the interval $[0, 3]$.

(a) Sketch the graph of the curve $y = f(x)$.

(b) What is the average value of $f(x) = 2x - x^2$ on $[0, 3]$?

(c) At what value or values of x in $[0, 3]$ is the average value of $f(x)$ attained?

2. (a) State *both parts* of the Fundamental Theorem of Calculus completely, including the hypotheses.

(b) Suppose $\int_0^x f(t)dt = \sin(x)$. Find $f(x)$.

3. Evaluate the following:

(a) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}dx.$

(b) $\displaystyle\frac{d}{dx}\int_1^{x^2} \sec(t-1)dt.$

4. Consider $f(x) = 3x^2$ and a partition of the interval $[0, 1]$ into **two** subintervals of equal length.

(a) Using the left endpoints of each subinterval to evaluate f, approximate $\int_0^1 3x^2dx$ by the Riemann sum $R = \sum_{k=1}^{n} f(c_k)\Delta x$.

(b) Use the trapezoidal rule $T = \frac{h}{2}\left[y_0 + 2\left(y_1 + y_2 + \ldots + y_{n-1}\right) + y_n\right]$ with the same two subintervals to approximate $\int_0^1 3x^2dx$.

(c) Is R or T a better estimate for $\int_0^1 3x^2dx$? Explain.

(d) How many subintervals of $[0, 1]$ would be enough to be sure that the trapezoidal rule for $f(x) = 3x^2$ would have an error less than or equal to 0.5×10^{-4}, given that $|E_T| \leq \frac{b-a}{12}h^2M$?

5. (a) Find $f^{-1}(x)$ when $f(x) = x^2 - 2x + 1, x \geq 1$.

(b) What are the domain and range of the $f^{-1}(x)$ found in part (a)?

(c) Find y' when $y = \dfrac{\ln x}{1 + \ln x}$.