APPM 1350 FINAL EXAM FALL 1999

On the front of your bluebook, please write: a grading key, your name, student ID, and time of the lecture you are registered in. This exam is worth 200 points and has 6 questions, written on **both** sides of this paper. **Show your work!**

1. (30 points) Find the requested information:

$$(a)\quad \int \frac{2x}{1+x^2}\,dx \qquad\qquad (b)\quad \int_0^1 \frac{2}{1+x^2}\,dx$$

$$(c)\quad \frac{d}{dx}\int_1^{e^{2x}} \ln t\; dt \qquad\qquad (d)\quad \frac{dy}{dx} \text{ for } x = 2^{-y}$$

2. (30 points) Find the requested limit, if it exists. If no limit exists, explain why.

$$(a)\quad \lim_{x\to\infty} \frac{\sqrt{9x+1}}{\sqrt{x+1}} \qquad\qquad (b)\quad \lim_{x\to 2} \frac{x}{x-2}$$

$$(c)\quad \lim_{x\to 1^+} x^{1/(1-x)} \qquad\qquad (d)\quad \lim_{x\to 0} \frac{\int_0^x f(t)\,dt}{x}$$

3. (30 points) Sand falls from a conveyor belt at the rate of $10m^3/min$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the (a) height and (b) radius changing when the pile is 4 m high? (Note: Be sure to use correct units in your answer. The volume of a cone is given by $V = (1/3)\pi r^2 h$.)

4. (40 points) Let $f(x) = \dfrac{\ln x}{1+\ln x}$.

   (a) What is the natural domain, D, of f? (Note: The natural domain is the largest set on which the function is defined.)

   (b) Compute $f'(x)$.

   (c) Does $f'(x)$ ever change sign in D? If so, where?

   (d) What asymptotes does $f(x)$ have? (Hint: There are no oblique asymptotes.)

   (e) Graph $y = f(x)$ together with its asymptotes, showing any maxima or minima that it might have.

Formulas you may need (a is a nonzero constant):

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \text{ valid when } u^2 < a^2$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \text{ valid for all } u$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \text{ valid for } u^2 > a^2$$

5. (35 points) Do 5 of the following TRUE/FALSE questions. Clearly indicate in your bluebook which 5 you are doing. If the statement is true, write TRUE and explain why it is true. If it is false, write FALSE and explain why it is false or give an example to show that it is false.

   (a) $\frac{d}{dx}\sin^2(\pi/4) = 2\sin(\pi/4)\cos(\pi/4)$
   (b) If $\lim_{x\to a} f(x)$ exists then f is continuous at $x = a$.
   (c) If $f(x)$ is continuous at $x = a$, then $\lim_{x\to a} f(x)$ exists.
   (d) If $y = f(x)$ is concave up on $[a, b]$ then $f(x)$ is increasing on $[a, b]$.
   (e) If $f'(a) = 0$, then the linearization of f about $x = a$ is constant.
   (f) If $\int f(x)\, dx = C$, where C is a constant, then $f(x) = 0$ everywhere.
   (g) If $\int_a^b f(x)\, dx > 0$ then f is positive somewhere in $[a, b]$.

6. (35 points)

   (a) Let $f(x) = x^3 - x^2 + 3$ with $x_0 = 1$. Find x_1 and x_2 using Newton's method.
   (b) Choose the graph that best matches each of the following 5 functions. For this question, you need not show any work. In your bluebook, your answer should be of the form: $y_1 - A$, $y_2 - B$, etc.

      (1) $y_1 = \ln x$ (2) $y_2 = e^{-x}$ (3) $y_3 = \sin^{-1} x$ for $-1 \le x \le 1$

      (4) $y_4 = 1/x$ for $x > 0$ (5) $y_5 = e^x$