

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, and (3) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (20 points) Evaluate the following and be sure to show all of your work.

(a) Find $\frac{df}{dx}$ where $f(x) = (\sin x)^{\ln x}$

(b) Find $\frac{df}{d\theta}$ where $f(\theta) = \frac{\theta \sin \theta}{\sqrt{\sec \theta}}$

(c) Find the value of x that maximizes $f(x)$ where $f(x)$ defined as $f(x) = \int_0^x t(1-t) dt$ without directly evaluating the integral.

(d) If $f(x) = x^3 - 1$, compute $f^{-1}(x)$ and sketch both $f(x)$ and f^{-1} on the same set of axis.

2. (20 points) Evaluate the following integrals and show all of your work.

(a) Find $\int \cos^2 x dx$

(b) Find $\int (\cos x) e^{\sin x} dx$

(c) Find $\int_{-1}^1 3x^2 \sqrt{x^3 + 1} dx$

3. (20 points) Consider the function $y = x^3 - 3$ on the closed interval $[0, 1]$.

(a) Sketch the graph of $f(x)$.

(b) Determine the mean value of $f(x)$ on the interval $[0, 1]$.

4. (20 points) Consider the integral $\int_0^1 x^3 dx$.

(a) Use the Trapezoidal rule $T = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$ with $n = 6$ and $h = \frac{b-a}{n}$ to approximate the integral. You do not need to find the value of the sum.

(b) Use the error estimate $|E_T| \leq \frac{(b-a)}{12} h^2 M$ where $M = \max |f''(x)|$ on the interval $[a, b]$ to estimate the error.

5. (20 points) Consider the differentiable function $f(x)$ defined on $[0, 4]$. You know the following information about the function: $f(1) = 2$; $f(4) = 5$; $f'(1) = 3$; $f'(4) = 0$. Determine the following:

(a) $f^{-1}(2)$

(b) $f^{-1}(5)$

(c) $\frac{df^{-1}}{dx}$ at $x = 2$

(d) $\frac{df^{-1}}{dx}$ at $x = 5$