

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name (Biswas, Norris) (4) a grading table. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Textbooks and class notes are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (25 points) Evaluate each of the following limits, if the limit exists. If the limit does not exist, state this. Explain your reasoning in each case! The correct answer with no supporting work is worth zero points,

(a) $\lim_{x \rightarrow \infty} (\ln(x) - \ln(\sin x))$

(b) $\lim_{x \rightarrow \infty} x^{3/x}$

(c) $\lim_{h \rightarrow 0} \frac{\int_0^{\pi+h} \cos(\sin t) dt - \int_0^{\pi} \cos(\sin t) dt}{h}$

2. (25 points) Evaluate $\frac{dy}{dx}$ for the functions given below.

(a) $y = \tan(\cos^{-1}(x))$

(b) $x^2 + \cos(xy) = 0$

(c) $y = \int_0^{x^3} (\sqrt{t} + 1) dt$

3. (25 points) Evaluate the following integrals. Show all work.

(a) $\int \frac{-1}{\ln^2(\sin x)} \frac{\cos x}{\sin x} dx$

(b) $\int_{\pi/4}^{\pi/2} \cos(x)(\sin^3(x) + 1) dx$

(c) $\int_2^3 \frac{dx}{x^2 + 2x + 1}$

(d) $\int \frac{dx}{x^2 + 4x + 5}$

4. (25 points) Consider the function $y = x^4 - 4x^3 + 10$ on the closed interval $[-1, 4]$.

(a) Find all critical points.

(b) Determine all the local maximum and minimum values of the function.

(c) Determine the absolute maximum and minimum values.

(d) Are there any inflection points?

(e) Determine the equation of the line normal to the curve at the point $(1, 7)$.

5. (25 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 meters of wire at your disposal, what is the largest area you can enclose?

6. (25 points) A boyfriend is filming his girlfriend's bungee jump from a bridge. During the free-fall portion of the jump, her position below the bridge, s in feet, may be described by $s = -16t^2$, where time t is measured in seconds. The boyfriend and his camera are on the bridge, 200 feet away from the drop location. Find the rate of change in the angle of the camera (measured down from the horizontal) after she has free-fallen for $5/\sqrt{2}$ seconds.

7. (25 points) A colony of bacteria is grown under ideal conditions in a pizza box in a student's dormroom. Assume the population increases exponentially with time. At the end of 3 hours there are 10,000 bacteria. At the end of 5 hours there are 40,000.

(a) Find the rate constant k .

(b) How many bacteria were initially deposited on the pizza when the roommate sneezed on it just after delivery.

8. (25 points) Determine whether each of the following statements is true or false. For each statement, if it is true, give a reason why it is true. If it is false, either give a reason why it is false, or give a counterexample showing that it is false.

(a) Since $\lim_{x \rightarrow c} \frac{x^2 - 9}{x - 3}$ exists for all real numbers c , the function $f(x) = \frac{x^2 - 9}{x - 3}$ is continuous for all real numbers.

(b) If f is an even function and $\lim_{x \rightarrow 3^-} f(x) = 3$ then $\lim_{x \rightarrow 3^+} f(x) = 3$.

(c) Since $\sin x$ is continuous everywhere, then $\frac{1}{\sin x}$ is continuous everywhere.

(d) x grows slower than $x + \ln x$