

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (20 points) Evaluate each of the following and show all supporting work. If a limit does not exist, clearly state that fact, and explain your reasoning.

(a) $\lim_{x \rightarrow 9} \sqrt{2 + \sqrt{4x}}$

(d) $\lim_{h \rightarrow 0} \frac{(x+h)^{100} - x^{100}}{h}$ (Think derivative!)

(b) $\lim_{x \rightarrow -5} \frac{\frac{1}{5} + \frac{1}{x}}{x + 5}$

(e) $\lim_{x \rightarrow 0} x^2 \sin^2\left(\frac{1}{x}\right)$

(c) $\lim_{x \rightarrow 1} \frac{x + 1}{1 - x^2}$

2. (20 points) Find $\frac{dy}{dx}$ for each of the following functions using the appropriate differentiation rules. After you have taken the derivative, do not simplify your results!

(a) $y = \tan \pi + x^{-3}$

(b) $y = (x^9 + x^4 + 2x^3 + 8) \cdot (x^4 + 5x^3 + 7x^2 + 3x + 2)$

(c) $y = \frac{(x-1)(x-4)}{(x-2)}$

3. (20 points) Consider the function $f(x) = \sqrt{x}$.

(a) State the domain D of $f(x)$.

(b) Starting from the definition for the derivative of $f(x)$, show that $f'(x) = \frac{1}{2\sqrt{x}}$.

(c) Are there any points in D for which $f(x)$ is not differentiable? Explain your answer.

4. (20 points) The position S of a particle at time t is given by $S(t) = t^3 - 7t + 4$, where S is measured in meters and t is measured in seconds.

(a) Find the velocity of the particle when $t = 3$ seconds.

(b) Find the acceleration when $t = 3$ seconds.

(c) Determine the average velocity over the time interval $[0, 3]$ seconds.

(d) Suppose one were to plot $S(t)$ as a function of time. (You do not need to plot this!) Calculate the equation of the tangent line to the curve at the point corresponding to $t = 3$ seconds.

HEY, THERE'S MORE—TURN THE PAGE OVER!

5. (20 points) Consider a function $y = f(x)$ for which the following is known:

- $f(0) = 0$
- $\lim_{x \rightarrow 0^-} f(x) = 0$
- $\lim_{x \rightarrow 0^+} f(x) = 0$
- $\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} = -1$
- $\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} = 1$

Using the above information, answer the following questions. Be sure to explain your reasoning.

- (a) Is $x = 0$ in the domain of f ?
- (b) Does $\lim_{x \rightarrow 0} f(x)$ exist?
- (c) Is $f(x)$ continuous at $x = 0$?
- (d) Is $f(x)$ differentiable at $x = 0$?