APPM 1350 EXAM #3 Fall 2000

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (25 points) Evaluate each of the following integrals and show your work.

   (a) $\int \left(e^x + x^{-3} + x^{1/3} + \sec^2(x)\right) dx$

   (b) $\int x \sin(x^2)\sqrt{\cos(x^2)}\, dx$

   (c) $\int_2^5 \left(\frac{1}{x^2} + \sin(\pi x)\right) dx$

   (d) $\int_0^1 (x^4 + x + 1)^{23}(4x^3 + 1)dx$

2. (25 points) Evaluate each of the following derivatives and show your work.

   (a) $\frac{d}{dx}\left(\int_7^x t\cos(t^3)dt\right)$

   (b) $\frac{d}{dx}\left(e^{(x^2)}\ln(3x)\right)$

   (c) $\frac{d}{dx}\left(\frac{(x-1)(x-2)^2(x-3)^3(x-4)^4}{x}\right)$ (Hint: logarithmic differentiation)

   (d) $\frac{d}{dt}(f^{-1}(t))$ at $t = 2$ given that $f(2) = 3$, $f(5) = 2$, $f'(2) = 7$, and $f'(5) = 11$.

3. (20 points) We wish to approximate the integral of the function $f(x) = 4x^2 + 1$ on the interval $[0, 2]$.

   (a) Compute a Riemann sum approximation to this integral by dividing $[0, 2]$ into four equal subintervals and using the left endpoint of each subinterval for evaluation.

   (b) Compute the Trapezoidal rule approximation to this integral using four subintervals. (Hint: $T = \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$)

   (c) Compute $\int_0^2 f(x)dx$. Which approximation was better?

HEY, THERE'S MORE—TURN THE PAGE OVER!

4. (20 points) Suppose f and g are continuous functions with the following properties:

$f(0) = 2$	$f(1) = 0$	$f(2) = 1$
$g(0) = 1$	$g(1) = 2$	$g(2) = 0$
$\int_0^1 f(x)dx = \pi$	$\int_1^2 f(x)dx = \pi^3$	$\int_2^3 f(x)dx = \pi^5$
$\int_0^1 g(x)dx = \sqrt{2}$	$\int_1^2 g(x)dx = \sqrt{3}$	$\int_2^3 g(x)dx = \sqrt{5}$

Evaluate the following. You do not need to show your work, but box in your final answer. If one cannot be evaluated with the given information, write "NOT ENOUGH INFORMATION."

(a) $\int_0^1 f(r)dr$

(b) $\int_0^3 f(x)dx$

(c) $\int_3^2 g(x)dx$

(d) $\int_6^6 f(x)dx$

(e) $\int_1^2 (5f(x) + g(x))dx$

(f) $\int_0^1 f(x)g(x)dx$

(g) $\int_1^2 f(g(x))dx$

(h) $\int_2^3 f(g(x))f'(x)dx$

(i) $\int_1^0 f(g(x))g'(x)dx$

(j) $\int_0^{14} f(x)dx - \int_2^{14} f(x)dx$

5. (10 points)

(a) So far we do not know how to evaluate $\displaystyle\int_0^3 \sqrt{9 - x^2}\, dx$. Interpret this integral in geometrical terms, and use your interpretation to find the value of the integral.

(b) Suppose f is a continuous, increasing function on $[a, b]$. Find lower and upper bounds on $\displaystyle\int_a^b f(x)dx$ in terms of a, b, $f(a)$, and $f(b)$. (Hint: Draw a picture.)