APPM 1350 **FINAL EXAM** **SPRING 2001**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name, and (4) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (25 points) Evaluate the derivative $\dfrac{df}{dx}$ for each of the following functions.

   (a) $f(x) = \displaystyle\int_0^{\cos x} \sin t \, dt$

   (b) $f(x) = \sqrt{\sin(x^2)}$

   (c) $f(x) = \tan^{-1}(\ln x)$

   (d) $f(x) = \left(e^{(x^2+3x)}\right)^{5x}$

2. (25 points) Evaluate each of the following integrals and show your work.

   (a) $\displaystyle\int \frac{ds}{1+e^{-s}}$

   (b) $\displaystyle\int \frac{4\,ds}{\sqrt{4-s^2}}$

   (c) $\displaystyle\int_0^3 xe^{3x^2}\,dx$

   (d) $\displaystyle\int_1^e \frac{dx}{x(1+\ln x)}$

3. (25 points) Determine the following limits:

   (a) $\displaystyle\lim_{x\to\infty} \frac{1}{x^2}\int_0^x \sin^2(t)\,dt$

   (b) $\displaystyle\lim_{h\to 0} \frac{\sin^{-1}(h) - \sin^{-1}(0)}{h}$

   (c) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x^2}$

   (d) $\displaystyle\lim_{x\to 0} \frac{\sqrt{x} + \ln x}{x}$

4. (25 points) A water cup in the shape of a right circular cone is to be constructed by removing the circular sector from a circular sheet of paper of radius a and then joining the two straight edges of the remaining paper. Find the dimensions of the cup with the largest volume that can be constructed. Hint: the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the base and h is the height.

5. (25 points) Assume the rate at which a block of salt dissolves in water is directly proportional to the amount of salt that remains undissolved in the block. If a 10 pound block of salt is placed in a container of water and in 20 minutes there are 6 pounds of solid salt left, how long will it take for two more pounds to dissolve?

6. (25 points) Consider the function $f(x) = x(x-2)^2$.

   (a) Find any local maxima and minima.

   (b) Identify the intervals on which the function is increasing, and decreasing.

   (c) Locate any inflection points.

   (d) Identify the intervals on which the function is concave up and concave down.

   (e) Graph the function and clearly identify the features from parts (a) through (d).

7. (25 points) For each of the following statements, clearly mark either TRUE or FALSE in your bluebook. There is no partial credit, and credit will be given only for the correct BOXED answer.

   (a) If $f'(x) = 0$ then x is the location of a local extremum.

   (b) The function $f(x) = x^2 + 1$ is not invertible for $x \geq 0$.

(c) The function $f(x) = \ln(x^2 - 1)$ can be defined for all $x \geq 0$.

(d) If $\lim_{x \to c^-} f(x) = \lim_{x \to c^+}$ then $f(x)$ is continuous at $x = c$.

(e) The function $f(x) = \dfrac{(x+1)(3x^2+2)}{(x^2-1)}$ has two vertical asymptotes.

(f) The linearization of the function $f(x) = 5 + \cos x$ at the point (0,6) is the line $y = 5$.

A short table of integrals.

1. $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$ for $u^2 < 1$
2. $\displaystyle\int \frac{du}{1+u^2} = \tan^{-1}(u) + C$
3. $\displaystyle\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + C$ for $u^2 > 1$