APPM 1350 — Exam #3 — April 17, 2002

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Dougherty or Panayotaros) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. Please start each new problem on a new page.

1. (21 points) Find the requested information.
   (a) Let $g(x) = \int_1^x \frac{3t + \cos 4t}{15 + \sin^2 t}$. Compute $g'(x)$.
   (b) Let $f(x) = \ln(x^4 + \sin^2 x)$. Compute $f'(x)$.
   (c) If $v = \frac{ds}{dt} = \pi^2 \cos(\pi t)$ for all t, $s(0) = 2$, find $s(t)$.

2. (24 points) Calculate the following definite and indefinite integrals, as indicated.

$$\text{(a)} \int_1^4 \frac{x^{3/2} + 2}{\sqrt{x}}\, dx \qquad \text{(b)} \int x \tan(x^2 + 5)\, dx \qquad \text{(c)} \int_1^9 \frac{\ln x}{x}\, dx$$

3. (20 points) Consider the function $f(x) = x + \frac{1}{2} \sin x$.
   (a) Show that $y = f(x)$ is increasing for all real x.
   (b) Find the average of f over the interval $[0, 2\pi]$.
   (c) Find the area between the graph of f and the interval $[0, 2\pi]$ in the $x-$axis. You should discuss the sign of $y = f(x)$ for x in $[0, 2\pi]$.

4. (15 points)
   (a) Let $f(x) = \int_1^{\sqrt[3]{x}} \ln t\, dt$. Find $f'(8)$.
   (b) Evaluate the sum $\sum_{k=1}^{20} (1 + k)^2$. Use $\sum_{k=1}^{n} k = \frac{n(n + 1)}{2}$ and $\sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}$.
   (c) Find $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k^4}{n^5}$ by recognizing that the expression equals a definite integral and then evaluating the integral.

5. (20 points) Oil is leaking out of a tanker damaged at sea at a rate $L(t)$ (in gallons/hour). We measure the leakage rate $L(t)$ at different times as shown below. Assume that $L(t)$ is a continuous function of t.

Time, t (hours)	0	1	2	3	4
Leakage, $L(t)$ (gal/hr)	50	75	100	125	200

   (a) Write a definite integral involving $L(t)$ that would give us the total amount of oil leaked from $t = 0$ to $t = 4$.
   (b) Give an upper and a lower estimate of the total quantity of oil that has escaped after 4 hours, assuming that $L(t)$ is an increasing function of time t.
   (c) Use the trapezoidal rule to approximate the total quantity of oil that has escaped after 4 hours. (Hint: $T = \frac{h}{2}(y_0 + 2y_1 + \cdots + 2y_{n-1} + y_n)$.