

1. (20 points) Indicate whether the following are ALWAYS TRUE or FALSE

(a) $\lim_{x \rightarrow c}(a + b) = \lim_{x \rightarrow c}(a) + \lim_{x \rightarrow c}(b)$

(b) $\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$

(c) $\tan(a + b) = \tan(a) + \tan(b)$

(d) $\tan^{-1}(a + b) = \tan^{-1}(a) + \tan^{-1}(b)$

(e) $f(a + b) = f(a) + f(b)$

(f) $e^{a+b} = e^a + e^b$

(g) $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

(h) $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

(i) $\frac{d^2}{dx^2}(f(x) + g(x)) = \frac{d^2}{dx^2}f(x) + \frac{d^2}{dx^2}g(x)$

(j) $\ln(a + b) = \ln(a) + \ln(b)$

(h) $(a + b)^2 = a^2 + b^2$

2. (20 points) When the space shuttle is launched, the rockets generate a constant acceleration, k . The acceleration must be sufficient to reach the escape velocity of 10,000 km/hr over a trajectory of 20,000 km. Calculate the necessary acceleration, k based on an initial position of 0 and an initial velocity of 0.

3. (20 points) A rocket, rising vertically, is tracked by a radar station that is on the ground 4 miles from the launchpad. How fast is the rocket rising when it is 3 miles high and its distance from the radar station is increasing at a rate of 600 miles/hour?

4. (30 points) Calculate $\frac{dy}{dx}$ for the following:

(a) $y = x \sin\left(\frac{1}{x}\right)$

(b) $y = 3 \cot^2(x)$

(c) $y = 5 \cos^{-1}(\pi)$

(d) $y = \sin^{-1}(x^3)$

(e) $y = \frac{1 - x^2}{1 - x}$

(f) $y = (\sec(x) + \tan(x))(\sec(x) - \tan(x))$ simplify!

(g) $y = x^x$

5. (20 points) Let $f(x) = (x - a)^2 - b$ where a, b are positive.

(a) What is the domain and range of f .

(b) Show that the f is not one-to-one on $(-\infty, \infty)$.

(c) Find the smallest value of k such that f is one-to-one on $[k, \infty)$.

(d) Find $f^{-1}(x)$ for $f(x)$ with $x \geq k$.

(e) What is the domain and range of $f^{-1}(x)$.