

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (25 points) Evaluate the following integrals.

(a) $\int \frac{2}{5} \sec \theta \tan \theta d\theta$

(b) $\int_1^4 \frac{1}{\sqrt{x}(1 + \sqrt{x})^2} dx$

(c) $\int_0^\pi \sin^2 \theta d\theta$

(d) $\int \frac{4x}{1 + x^2} dx$

(e) $\int \cos(\sin(\sin(x))) \cos(\sin(x)) \cos(x) dx$

2. (20 points) Consider the function defined by $f(x) = \int_0^x t(a - t) dt$.

- (a) Without evaluating the integral, find the maximum value of $f(x)$ for $x \geq 0$.
(b) Find the linearization of $f(x)$ near $x = 0$.

3. (20 points)

- (a) Write down the general formula for Newton's method.
(b) Starting with $x_0 = 1$, use Newton's method to approximate the positive root of the polynomial $2x^2 - 3$ up to x_2 .

4. (20 points) We wish to approximate the integral of $f(x) = \sin(x)$ over the interval $[-\pi, \pi]$.

- (a) Compute the trapezoid rule approximation to the integral using 4 equally spaced subintervals.
(b) What is the upper limit for the error given $|E_T| \leq \frac{b-a}{12} h^2 M$?
(c) What is the actual error in the trapezoid rule approximation?

5. (15 points) State whether the following statements are TRUE or FALSE.

Warning! You will get +3 points for each CORRECT answer, and -1 point for each WRONG answer, so don't randomly guess!

(a) If $|f(x)| \geq |g(x)|$ on $[a, b]$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.

(b) All continuous functions on the interval $[0, 1]$ are integrable.

(c) All integrable functions on the interval $[0, 1]$ are continuous.

(d) The function $x^2 + 1$ may be integrated exactly using Simpson's rule with 2 subintervals.

(e) $f(f^{-1}(x)) = 1$