You must work **ALL the problems on the front** side of the exam. Work **4 out of 5 problems from the back**. Make an X in the grading table for the problem from the back that you don't want us to grade. Otherwise, we will grade the first 4 problems from the back.

Show ALL of your work in your bluebook and BOX in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, calculators and crib sheets are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (25 points) Indicate whether the following are ALWAYS TRUE or FALSE.

   (a) $\lim_{x\to c}\Big(f(x)+g(x)\Big) = A+B$ if $\lim_{x\to c} f(x) = A$ and $\lim_{x\to c} g(x) = B$.

   (b) $\tan^{-1}(a+b) = \tan^{-1}(a) + \tan^{-1}(b)$

   (c) $\lim_{x\to -2^-} f(x) = 3$ if we know that $\lim_{x\to 2^+} f(x) = 3$ and $f(x)$ is an even function.

   (d) $\int_a^b (f(x))^2\, dx = \left(\int_a^b f(x)\, dx\right)^2$

   (e) $\int_{-7}^{7} f(x)\, dx = 0$ where $f(x)$ is an odd function and $f(x)$ is integrable.

2. (25 points) Evaluate the following limits if they exist, otherwise, state DNE. Show your work!

   (a) $\lim_{x\to 2} \dfrac{x-2}{x^2-5}$

   (b) $\lim_{x\to\infty} \left(1+\dfrac{1}{x}\right)^x$

   (c) $\lim_{x\to\infty} \tan^{-1} x$

   (d) $\lim_{x\to 0} \left(\dfrac{3}{3x-x^2} - \dfrac{1}{x}\right)$

3. (25 points) Calculate dy/dx for each of the following:

   (a) $y = \int_1^{e^x} \ln(t)\, dt$

   (b) $y = \tan^{-1}(\ln x)$

   (c) $y = x^{\sqrt{x}}$

   (d) $y = (x^2-5x)^{22}(x^5-7)^{11}(x-x^3)^{15}$

4. (25 points) Evaluate the following integrals:

   (a) $\int \dfrac{dx}{(1+x)\,\ln(1+x)}$

   (b) $\int_0^{2\pi} |\sin x|\, dx$

   (c) $\int e^x(1+\cos e^x)\, dx$

   (d) $\int_0^1 \dfrac{x^3}{4+x^8}\, dx$

   (e) $\int \dfrac{dx}{e^x\sqrt{e^{2x}-1}}$

5. (25 points) Find all points on the curve $y = \sqrt{x}$ for $0 \le x \le 3$ that are closest to, and the points on the curve that are the greatest distance from, the point $(2, 0)$.

6. (25 points) A rocket, rising vertically, is tracked by a radar station that is on the ground 4 miles from the launch pad. How fast is the rocket rising when it is 3 miles high and its distance from the radar station is increasing at a rate of 600 miles/hour?

7. (25 points) The curve described by $x^3 + y^3 - 2xy = 0$ can be used to define a function $y = f(x)$ near the point $(1, 1)$.

   (a) Calculate dy/dx.

   (b) Determine the tangent line to curve at the point $(1, 1)$.

   (c) Determine the normal line to the curve at the point $(1, 1)$.

   (d) Find the linearization of $f(x)$ at the point $(1, 1)$.

8. (25 points) A cup of coffee is taken outside where the temperature is 0°C. After 10 minutes the temperature of the coffee is 30°C, and after 20 minutes the coffee's temperature is 15°C.

   (a) Use Newton's law of cooling to estimate the coffee's temperature when it was first taken outside.

   (b) When the coffee has been outside for 30 minutes, what is its temperature?

9. (25 points) Consider the function $f(x) = x^4 - 4x^3$ on the interval $[0, 5]$.

   (a) Find the roots of f.

   (b) Find and classify all local and absolute extrema of f.

   (c) Determine the location of any inflection points of f.

   (d) Sketch the graph of f, including all roots, relative extrema, and inflection points.

Newton's law of cooling

$$T - T_S = (T_0 - T_S)e^{-kt}$$

Volumes and surface areas

$$S_{\text{sphere}} = 4\pi r^2 \qquad V_{\text{sphere}} = \frac{4}{3}\pi r^3 \qquad S_{\text{cone}} = \pi r\sqrt{r^2 + h^2} \qquad V_{\text{cone}} = \frac{\pi}{3}\pi h r^2$$

Sum of angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad \cos(A + B) = \cos A \cos B - \sin A \sin B$$

Double angle relations and law of cosines

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2} \qquad c^2 = a^2 + b^2 - 2ab\cos\theta$$

Integrals

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \text{(valid for } u^2 < a^2\text{)}$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{|a|}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad \text{(valid for } u^2 > a^2\text{)}$$