ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your instructor's name, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (20 points) Multiple Choice (no justification needed for this problem)

   (a) Which of the following functions grows at the fastest rate as $x \to \infty$?
   (i) x^3 (ii) 5^x (iii) $\ln(x^3)$ (iv) e^x

   (b) If $\frac{dy}{dx} = \cos\left(\frac{x}{2}\right)$ and $y(\pi) = 1$ then $y =$
   (i) $\frac{1}{2}\sin(\frac{x}{2}) + \frac{1}{2}$ (ii) $-\frac{1}{2}\sin(\frac{x}{2}) + \frac{3}{2}$ (iii) $2\sin(\frac{x}{2}) - 1$ (iv) $-2\sin(\frac{x}{2}) + 3$

   (c) $f(x)$ will have a local minumum at $x = a$ if
   (i) $f'(a) > 0$ and $f''(a) > 0$ (ii) $f'(a) = 0$ and $f''(a) > 0$ (iii) $f'(a) = 0$ and $f''(a) < 0$ (iv) $f'(a) = 0$ and $f''(a) = 0$

   (d) The number $\cos^{-1}(\log_2 8 - e^{2\ln\sqrt{2}})$ is equal to
   (i) 0 (ii) e (iii) $\pi/2$ (iv) 1

2. (20 points) Evaluate the following limits if they exist, if they don't exist or are infinite please indicate so.

   (a) $\displaystyle\lim_{x\to\infty} \frac{\ln(e^x - 1)}{\ln(x)}$ (b) $\displaystyle\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}$

   (c) $\displaystyle\lim_{h\to 0} \frac{\sin(x+h) - sin(x)}{h}$
   (hint: think derivative)
   (d) $\displaystyle\lim_{x\to\infty} e^{-0.2x}$

3. (20 points) Calculate $\frac{dy}{dx}$ for each of the following:

   (a) $y = \tan\left(e^{x^2}\right)$ (b) $y = x^x$

   (c) $y = \dfrac{\sin(x)}{x}$ (d) $y = \displaystyle\int_1^{\ln(x)} e^{2t}\,dt$

4. (20 points) Evaluate the following integrals.

   (a) $\displaystyle\int \frac{\sec^2 x}{\sqrt{9 - \tan^2 x}}\,dx$ (b) $\displaystyle\int \frac{3x}{\sqrt{1-x^2}}\,dx$

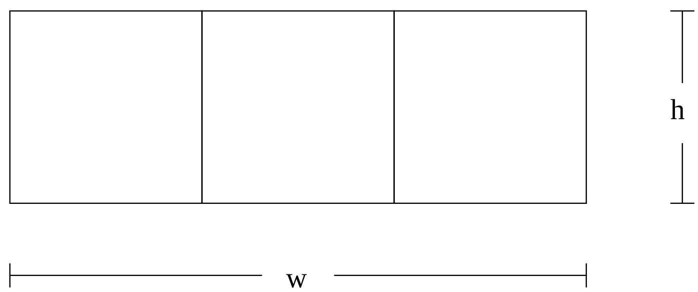
   (c) $\displaystyle\int_{1/2}^{1} \sec(\cos^{-1}(x))\,dx$
   (hint: rewrite $\sec(\cos^{-1}(x))$)
   (d) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \cos^3(x)\,dx$
   (hint: $\sin^2(x) + \cos^2(x) = 1$)

COOL, THERE'S MORE!! TURN THE PAGE OVER

5. (15 points) Find the linearization of $f(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt$ at $x = -1$.

6. (15 points) A spherical balloon is inflated by blowing in air at a rate of $100\pi\ in^3/min$. At what rate is the radius of the balloon changing when the radius is 10 in ?

7. (20 points) A rancher wants to fence in a rectangular region seperated into three individual regions as shown below. If he has only $200ft$ of fencing what is the maximum area he can enclose?

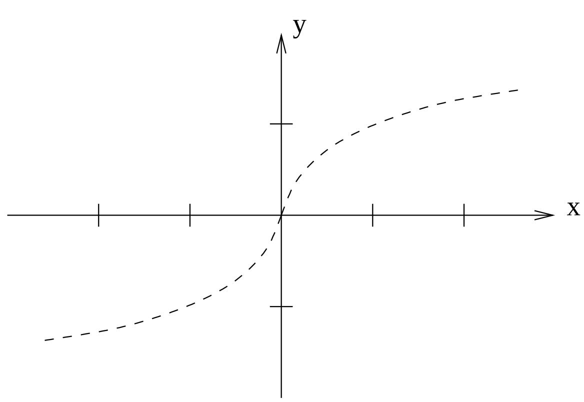


8. (20 points) The general formula for Newton's Method is given by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

(a) The function $f(x) = x^{\frac{1}{3}}$ as pictured below has a root at $x = 0$. Starting with an inital guess of $x_0 = 1$, perform 2 iterations of Newton's method.

(b) By substituting $f(x) = x^{\frac{1}{3}}$ into the formula for Newton's Method show that consecutive iterates will always satisfy the relation $x_{n+1} = -2x_n$.

(c) Explain why your result from part (b) shows that Newton's method, with an initial guess $x_0 \neq 0$, will not converge to the root at $x = 0$ for this problem.



The following may be useful:

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C\,,\ (u^2 < a^2) \qquad V_{sphere} = \tfrac{4}{3}\pi r^3$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C\,,\ (u^2 > a^2)$$