

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, and (3) a grading table.

Please start each problem on a new page. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes, crib sheets, and calculators are NOT permitted.

1. (10 points) Let  $f(x)$  be a function. You may describe, write down, tell a story, etc, to get your point across for the following questions.

- (a) Explain the Fundamental Theorem of Calculus as it relates to  $f(x)$ . What has to be true about  $f(x)$  for the FToC to be applicable? (note: there are two different versions of the FToC, you only need to discuss one)
- (b) Explain the Mean Value Theorem *it as it relates to integrals*. What has to be true about  $f(x)$  for the MVT to be applicable?

2. (30 points) Determine each of the following.

(a)  $\frac{d}{dx} \left( \int_{x^2}^0 \sin(t^2) \right) dt$       (b) the average value of  $f(t) = \frac{t}{(t^2 + 1)^2}$  on  $[0, 2]$

(c)  $\int_0^{\frac{1}{2}} \frac{9r^2}{\sqrt{1-r^3}} dr$       (d)  $\int_0^{\frac{\pi}{4}} \sin(2x) \cos(2x) dx$

(e)  $\int (4 - x^3)^2 dx$       (f)  $\int \frac{1 - \sqrt{u}}{\sqrt{u}} du$

3. (25 points) For each of the following statements, mark **True** if it is true. If it is false: 1. mark **FALSE** and 2. replace it with a correct statement or a counterexample.

(a)  $\int_1^1 f(x) dx = 1$       (b)  $\int_1^3 f(x) dx + \int_3^5 f(x) dx = \int_1^5 f(x) dx$

(c)  $\int_b^a f(x) dx = \int_a^b f(-x) dx$       (d)  $\sum_{k=2}^4 \frac{(-1)^{k-1}}{k-1} = \sum_{k=-1}^1 \frac{(-1)^k}{k+2}$

(e) on  $[a, b]$ ,  $\lim_{\|\Delta x \rightarrow 0\|} \sum_{k=1}^n f(c_k) \Delta x = \int_a^b f(x) dx$

COOL, THERE'S MORE!! TURN THE PAGE OVER

4. (20 points) A computer gives a digital readout of fuel consumption for a small aircraft in  $\frac{\text{gallons}}{\text{min}}$ . During a 30 minute trip, the following data was collected every 5 minutes:

Time (min)	0	5	10	15	20	25	30
$\frac{\text{gal}}{\text{min}}$	2	3	1	3	2	3	1

- (a) Use the Simpson Rule to get an estimate of the total consumption of gas (in gallons) during the trip.
- (b) The data in the table can be approximated by the function  
 $f(x) = 0.0001x^3 - 0.06x^2 + 0.5x + 2$ .  
 What is the error estimate for your calculations?
- (c) How many subintervals (n) would be needed to compute  $f(x)$  with an error at most 0.01 using the Simpson Rule?
5. (15 points) The acceleration of a particle moving along a straight line is given by  $a(t) = -6 \sin(2t) \frac{\text{feet}}{\text{sec}^2}$ , with an initial velocity  $v(0) = 4 \frac{\text{feet}}{\text{sec}}$  and initial position  $s(0) = -1 \text{ft}$ .
- (a) Find  $v(t)$
- (b) Find  $s(t)$
- (c) Find the location of the particle at  $t = 1$  sec. Do not simplify.

### Formulae

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

$$T = \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$

$$|E_T| \leq \frac{b-a}{12} h^2 M$$

$$S = \frac{h}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

$$|E_S| \leq \frac{b-a}{180} h^4 M$$