APPM 1350 — Exam #3 — November 19, 2003

On the front of your bluebook print (1) your name, (2) your student ID number, (3) your instructor's name (Carvalho, Dougherty, Norris, Robertson, Sprague) and (4) a grading table. Show all work in your bluebook and BOX IN YOUR FINAL ANSWERS where appropriate. A correct answer with no supporting work may receive no credit while an incorrect answer with some correct work may receive partial credit. Textbooks, class notes, calculators and crib sheets are not permitted. There a total of FIVE problems, on both sides of this paper. Please start each new problem on a new page.

1. (20 points) Consider the functions $g(x) = \sqrt{x}$ and $f(x) = x^2 - 5$.
   (a) Find the linearization of $g(x)$ near $x = 4$.
   (b) Use the linearization from part(a) to estimate the value of $\sqrt{5}$.
   (c) Use Newton's Method to write down an iteration formula ($x_{n+1} = ...$) in terms of x_n that can be used to find the solution of $f(x) = 0$.
   (d) Starting with an initial guess of $x_0 = 1$, use the iteration formula from part(c) to estimate the value of $\sqrt{5}$ with x_2.

   (Notice that in the problem above we used two different methods to estimate the value of $\sqrt{5}$.)

2. (30 points) Evaluate the following integrals.
   (a) $\int (2-x)^{3/5}\, dx$
   (b) $\int \sec(\pi)\tan(\frac{\pi}{4})\cos^2(\theta)\, d\theta$
   (c) $\int_0^1 \frac{6y^2}{(y^3+1)^3}\, dy$
   (d) $\int_4^9 \frac{1}{\sqrt{z}}\left(\frac{1-\sqrt{z}}{1-z}\right)^2 dz$ (Hint: simplify the integrand before attempting to integrate.)

3. (20 points) Find the requested information.
   (a) Find $y(x)$ that satisfies the initial value problem $\frac{dy}{dx} = \frac{1}{2}\sec(x)\tan(x)$, $y(0) = 1$.
   (b) Compute $\frac{d}{dx}\int_0^\pi \frac{\sin(t) - \cos(t)}{\sqrt{2+\cos(t)}}\, dt$.
   (c) Compute $\frac{d}{dx}\int_1^{x^2} \sec(t-1)\, dt$.
   (d) Find the inverse function of $f(x) = 64x^3 - 4$.

COOL, THERE'S MORE ON THE BACK!

4. (15 points) Consider the definite integral

$$\int_1^3 4^x \, dx$$

(a) Estimate the value of the integral by computing its Riemann Sum with 4 equally-spaced subintervals. Use the left-hand endpoint of each subinterval to compute the height of the corresponding rectangle.

(b) Use the Trapezoidal Rule with 4 equally-spaced subintervals to estimate the value of the integral.

(Notice that we haven't worked with functions of the type $f(x) = 4^x$ in this course so far. This type of function will be properly defined in chapter 6, and you'll even learn how to evaluate the integral of this problem exactly. By then, you'll know that $f(x) = 4^x$ is continuous, increasing and concave up in the interval [1,3], being able to conclude that you overestimated the value of the integral on part(b) and underestimated it on part(a).)

5. (15 points) Consider the function

$$g(x) = \int_1^x f(t) \, dt$$

If $f(t)$ has a **positive derivative** for all values of t and $f(2) = 0$, decide which of the following statements are **always true**, which are **always false** and which you have **not enough information** to make a claim. There's no need to justify your claims. Simply write on your blue book the statement letter along with your claim: TRUE, FALSE or NOT ENOUGH INFO.

(a) $g(x)$ is a differentiable function of x.

(b) $g(x)$ is a continuous function of x.

(c) The graph of $g(x)$ has a horizontal tangent at $x = 2$.

(d) $g(x)$ has a local maximum at $x = 2$.

(e) The graph of $g(x)$ has an inflection point at $x = 2$.